General Aptitude

Q.1 to Q.5 Carry ONE Mark Each

Question 1

If '$\rightarrow$' denotes increasing order of intensity, then the meaning of the words $[\text{drizzle} \rightarrow \text{rain} \rightarrow \text{downpour}]$ is analogous to $[\_\_\_\_\_\_ \rightarrow \text{quarrel} \rightarrow \text{feud}]$. Which one of the given options is appropriate to fill the blank ?

(A) bicker
(B) bog
(C) dither
(D) dodge

Ans. (A)

Question 2

Statements :

1. All heroes are winners.
2. All winners are lucky people.

Inferences :

I. All lucky people are heroes.
II. Some lucky people are heroes.
III. Some winners are heroes.

Which of the above inferences can be logically deduced from statements 1 and 2 ?

(A) Only I and II
(B) Only II and III
(C) Only I and III
(D) Only III

Ans. (B)

Question 3

A student was supposed to **multiply** a positive real number p with another positive real number q. Instead, the student **divided** p by q. If the percentage error in the student's answer is 80%, the value of q is

(A) 5
(B) $\sqrt{2}$
(C) 2
(D) $\sqrt{5}$

Ans. (D)

Question 4

If the sum of the first 20 consecutive positive odd numbers is divided by 20^2, the result is

(A) 1
(B) 20
(C) 2
(D) $\frac{1}{2}$

Ans. (A)

Question 5

The ratio of the number of girls to boys in class VIII is the same as the ratio of the number of boys to girls in class IX. The total number of students (boys and girls) in classes VIII and IX is 450 and 360, respectively. If the number of girls in classes VIII and IX is the same, then the number of girls in each class is

(A) 150 (B) 200

(C) 250 (D) 175

Ans. (B)

Q.6 to Q.10 Carry TWO Marks Each

Question 6

In the given text, the blanks are numbered (i)-(iv). Select the best match for all the blanks.

Yoko Roi stands ___(i)___ as an author for standing ___(ii)___ as an honorary fellow, after she stood ___(iii)___ her writings that stand ___(iv)___ the freedom of speech.

(A) (i) out (ii) down (iii) in (iv) for

(B) (i) down (ii) out (iii) by (iv) in

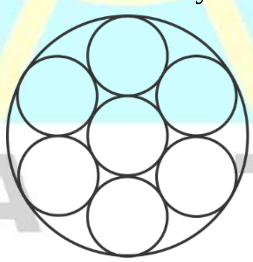
(C) (i) down (ii) out (iii) for (iv) in

(D) (i) out (ii) down (iii) by (iv) for

Ans. (D)

Question 7

Seven identical cylindrical chalk-sticks are fitted tightly in a cylindrical container. The figure below shows the arrangement of the chalk-sticks inside the cylinder.



The length of the container is equal to the length of the chalk-sticks. The ratio of the occupied space to the empty space of the container is

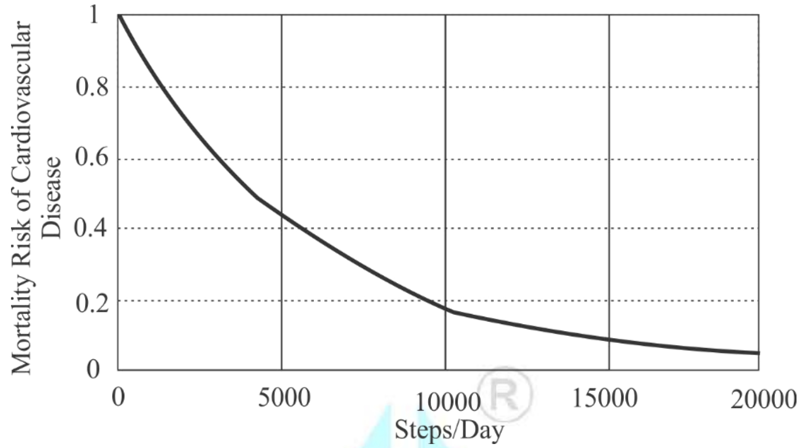
(A) 5/2 (B) 7/2

(C) 9/2 (D) 3

Ans. (B)

Question 8

The plot below shows the relationship between the mortality risk of cardiovascular disease and the number of steps a person walks per day. Based on the data, which one of the following options is true ?



(A) The risk reduction on increasing the steps/day from 0 to 10000 is less than the risk reduction on increasing the steps/day from 10000 to 20000.

(B) The risk reduction on increasing the steps/day from 0 to 5000 is less than the risk reduction on increasing the steps/day from 15000 to 20000.

(C) For any 5000 increment in steps/day the largest risk reduction occurs on going from 0 to 5000

(D) For any 5000 increment in steps/day the largest risk reduction occurs on going from 15000 to 20000

Ans. (C)

Question 9

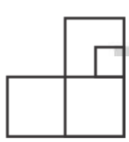
Five cubes of identical size and another smaller cube are assembled as shown in Figure A. If viewed from direction X, the planar image of the assembly appears as Figure B.

Figure A

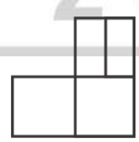
Figure B

If viewed from direction Y, the planar image of the assembly (Figure A) will appear as

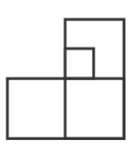
(A)



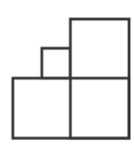
(B)



(C)



(D)



Ans. (A)

Question 10

Visualize a cube that is held with one of the four body diagonals aligned to the vertical axis. Rotate the cube about this axis such that its view remains unchanged. The magnitude of the minimum angle of rotation is

(A) 120^0 (B) 60^0

(C) 90^0 (D) 180^0

Ans. (A)

Technical Section

Q.11 to Q.35 Carry ONE Mark Each

Question 11

Let $z = x + iy$ be a complex variable and $\bar{z}$ be its complex conjugate. The equation $\bar{z}^2 + z^2 = 2$ represents a **(1 Mark)**

(A) parabola (B) hyperbola

(C) ellipse (D) circle

Ans. (B)

Sol. Given :

$$z = x + iy$$

$$\bar{z} = x - iy$$

$$(\bar{z})^2 = (x - iy)^2$$

$$= x^2 + (iy)^2 - 2ixy$$

$$\Rightarrow \quad (\bar{z})^2 = x^2 - y^2 - 2ixy$$

$$\Rightarrow \quad (z)^2 = (x + iy)^2$$

$$= x^2 + (iy)^2 + 2ixy$$

$$(z)^2 = x^2 - y^2 + 2ixy$$

The equation

$$(\bar{z})^2 + z^2 = 2$$

$$(x^2 - y^2 - 2ixy) + (x^2 - y^2 + 2ixy) = 2$$

$$2x^2 - 2y^2 = 2$$

$$x^2 - y^2 = 1$$

The above eqaution represents the equation of rectangular hyperbola.

Hence the correct option is (B).

Key Point

(i) $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$: Ellipse with centre at $(0,0)$ and major axis = 2a and minor axis = 2b

(ii) **Hyperbola :**

$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$: Hyperbola with vertex at $(a,0)$ and $(-a,0)$.

(iii) **Rectangular Hyperbola :**

- $x^2-y^2=a^2$
- $xy=a^2$

Question 12

The pressure drop across a control valve is constant. The control valve with inherent characteristic has decreasing sensitivity. If x represents the fraction of maximum stem position of the control valve, then the function $f(x)$ representing the fraction of maximum flow is

(A) α^{x-1}, where α is constant (B) $\sqrt{x}$

(C) x (D) x^2

Ans. (B)

Question 13

A discrete-time sequence is given by $x[n]=[1,2,3,4]$ for $0\le n\le 3$. The zero lag auto-correlation value of $x[n]$ is

(A) 1 (B) 10

(C) 20 (D) 30

Ans. (D)

Question 14

Match the following measuring devices with their principle of measurement.

Measuring Device		Principle of Measurement	
(P)	Optical pyrometer	(I)	Variation in mutual inductance
(Q)	Thermocouple	(II)	Change in resistance
(R)	Strain gauge	(III)	Wavelength of radiated energy
(S)	Linear variable differential transformer	(IV)	Electromotive force generated by two dissimilar metals

(A) (P) − (III), (Q) − (IV), (R) − (II), (S) − (I)

(B) (P) − (IV), (Q) − (III), (R) − (II), (S) − (I)

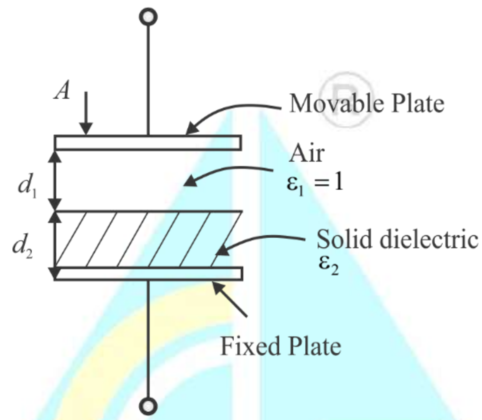
(C) (P) − (III), (Q) − (I), (R) − (IV), (S) − (II)

(D) (P) − (II), (Q) − (IV), (R) − (I), (S) − (III)

Ans. **(A)**

Question 15

The capacitor shown in the figure has parallel plates, with each plate having an area A. The thickness of the dielectric materials are d_1 and d_2 and their relative permittivities are ε_1 and ε_2, respectively. Assume that the fringing field effects are negligible and ε_0 is the permittivity of free space.



If d_1 is decreased by δd_1, the resultant capacitance becomes

(A) $\dfrac{\varepsilon_0 A}{d_1-\delta d_1+\dfrac{d_2}{\varepsilon_2}}$ (B) $\dfrac{\varepsilon_0 A}{d_2+\dfrac{d_1}{\varepsilon_2}}$

(C) $\dfrac{\varepsilon_0 A}{d_2-\delta d_2+\dfrac{d_1}{\varepsilon_2}}$ (D) $\dfrac{\varepsilon_0 A}{d_1+\delta d_1+\dfrac{d_2}{\varepsilon_2}}$

Ans. **(A)**

Question 16 **[Digital Electronics]**

Among the given options, the simplified form of the Boolean function $F=\left(A+\bar{A}.B\right)+\bar{A}.\left(A+\bar{B}\right).C$ is

(A) $A+B+C$ (B) $A.B.C$

(C) $B+\bar{A}.C$ (D) $\bar{A}+B.C$

Ans. **(A)**

Sol. **Given :** $F=\left(A+\bar{A}B\right)+A.\left(A+\bar{B}\right)\cdot C$

Distributive laws :

(i) $A+BC=\left(A+B\right)\left(A+C\right)$

(ii) $A+\bar{A}C=\left(A+\bar{A}\right)\left(A+C\right)$

$=1\left(A+C\right)$

$\therefore \quad A+\bar{A}=1 \quad$ (OR Laws)

$$F=(A+\bar{A})(A+B)+\bar{A}(A+\bar{B})C$$

$$F=(A+B)(\bar{A}A+\bar{A}\bar{B})C$$

$$F=(A+B)(0+\bar{A}\bar{B})C$$

$$F=A+B+\bar{A}\bar{B}C$$

$$F=A+B+\bar{A}\bar{B}C+\bar{A}\bar{B}C$$

$$F=A+\bar{A}(\bar{B}C)+B+\bar{B}(\bar{A}C)$$

$$F=A+\bar{B}C+B+\bar{A}C$$

$$F=(A+\bar{A}C)+(B+\bar{B}C)$$

$$F=(A+C)+(B+C)$$

$$F=A+B+C$$

Hence the correct option is (A).

Key points :

Distributive laws :

(1) $\quad A+BC=(A+B)(A+C)$

(2) $\quad A+\bar{A}BC=(A+\bar{A})(A+BC)$

(3) $\quad A+\bar{A}B=(A+\bar{A})(A+B)$

(4) $\quad A+\bar{A}=1 \quad$ (OR Laws)

(5) $\quad 1+$ Any Literal $=1$

Method (2):

$$F=(A+\bar{A}B)+\bar{A}(A+\bar{B})C$$

$$F=A+B+\bar{A}\bar{B}C$$

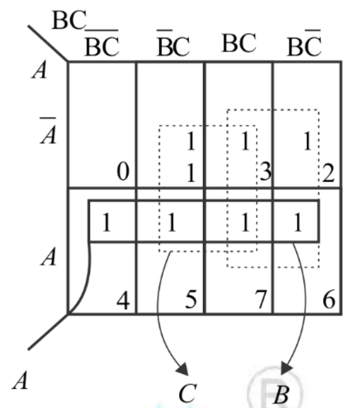
$$F=A(B+\bar{B})(C+\bar{C})+B(A+\bar{A})(C+\bar{C})+\bar{A}\bar{B}C$$

$$F=A\bar{B}\bar{C}+A\bar{B}C+AB\bar{C}+ABC+B\bar{A}\bar{C}+B\bar{A}C+BA\bar{C}+BAC+\bar{A}\bar{B}C$$

$$F=A\bar{B}\bar{C}+A\bar{B}C+AB\bar{C}+ABC+\bar{A}B\bar{C}+\bar{A}BC+\bar{A}\bar{B}C$$

$$F=A\bar{B}\bar{C}+A\bar{B}C+AB\bar{C}+ABC+\bar{A}B\bar{C}+\bar{A}BC+\bar{A}\bar{B}C$$

K-map for 3-variables.



Thus the SOP expression of function F is

$$F = A + B + C$$

Hence the correct option is (A).

Question 17 **[Control System]**

Consider the state-space representation of a system

$$\dot{x} = Ax + Bu$$

where x is the state vector, u is the input, A is the system matrix and B is the input matrix. Choose the matrix A from the following options such that the system has a pole at the origin.

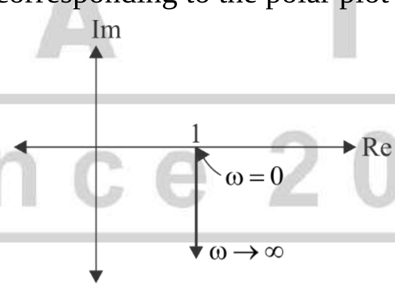
(A) $\begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$ (B) $\begin{bmatrix} 1 & -1.5 \\ -2 & 3 \end{bmatrix}$

(C) $\begin{bmatrix} 1 & 1.5 \\ 2 & -3 \end{bmatrix}$ (D) $\begin{bmatrix} 0 & 1 \\ -2 & 3 \end{bmatrix}$

Ans. (B)

Question 18

The sinusoidal transfer function corresponding to the polar plot shown in the figure, for $T > 0$, is



(A) $1 - j\omega T$ (B) $\dfrac{1 - j\omega T}{1 + j\omega T}$

(C) $1 + j\omega T$ (D) $\dfrac{1}{1 + j\omega T}$

Ans. (A)

Question 19

A matrix M is constructed by stacking three column vectors v_1, v_2, v_3 as

$$M = [v_1 \quad v_2 \quad v_3].$$

Choose the set of vectors from the following options such that $rank(M) = 3$. **(1 Mark)**

(A) $v_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, v_2 = \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}, v_3 = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$

(B) $v_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, v_2 = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}, v_3 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

(C) $v_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, v_2 = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}, v_3 = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$

(D) $v_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, v_2 = \begin{bmatrix} -1 \\ 1 \\ -1 \end{bmatrix}, v_3 = \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}$

Ans. **(C)**

Sol. **Given :** Matrix M is constructed by stacking three column vectors v_1, v_2, v_3 as

$$M = [v_1 \quad v_2 \quad v_3]$$

Rank $(M) = 3$

Checking from options

From option (A) :

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, v_2 = \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}, v_3 = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

Constructing matrix M by stacking three column vectors v_1, v_2, v_3

$$M = \begin{bmatrix} 1 & 0 & 1 \\ 0 & -1 & -1 \\ 1 & 0 & 1 \end{bmatrix}$$

Using elementary transformation

$$R_3 \to R_3 - R_1$$

$$M = \begin{bmatrix} 1 & 0 & 1 \\ 0 & -1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$

There exist two non zero rows in above matrix

So, rank $(M) = 2$

From option (B) :

$$v_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, v_2 = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}, v_3 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$M = [v_1 \quad v_2 \quad v_3]$$

$$M = \begin{bmatrix} 1 & -1 & 0 \\ 1 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

Using elementary transformation

$$R_3 \to R_3 + R_1$$

$$M = \begin{bmatrix} 1 & -1 & 0 \\ 1 & 0 & 0 \\ 2 & 0 & 0 \end{bmatrix}$$

Using elementary transformation

$$R_3 \to R_3 - 2R_2$$

$$M = \begin{bmatrix} 1 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

There exist two non zero rows in above matrix

So, $\text{rank}(M) = 2$

From option (C) :

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, v_2 = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}, v_3 = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

$$M = [v_1 \quad v_2 \quad v_3]$$

$$M = \begin{bmatrix} 1 & -1 & 1 \\ 0 & 0 & -1 \\ 1 & 1 & 1 \end{bmatrix}$$

Using elementary transformation

$$R_3 \to R_3 - R_1$$

$$M = \begin{bmatrix} 1 & -1 & 1 \\ 0 & 0 & -1 \\ 0 & 2 & 0 \end{bmatrix}$$

There exist three non zero rows in above matrix

So, $\text{rank}(M) = 3$

From option (D) :

$$v_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, v_2 = \begin{bmatrix} -1 \\ 1 \\ -1 \end{bmatrix}, v_3 = \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}$$

$$M = [v_1 \quad v_2 \quad v_3]$$

$$M = \begin{bmatrix} 1 & -1 & 0 \\ 1 & 1 & -1 \\ 1 & -1 & 0 \end{bmatrix}$$

Using elementary transformation

$$R_3 \to R_3 - R_1$$

$$M = \begin{bmatrix} 1 & -1 & 0 \\ 1 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$

There exist two non zero rows in above matrix

So, $\text{rank } (M) = 2$

Hence, the correct option is (C).

Question 20

The capacitance formed between two concentric spherical metal shells having radii x and y with $y > x$ is

Note : $\in$ is the permittivity of the medium between the shells.

(A) $4\pi \in \left(\dfrac{xy}{y-x}\right)$

(B) $4\pi \in \left(\dfrac{x^2}{y-x}\right)$

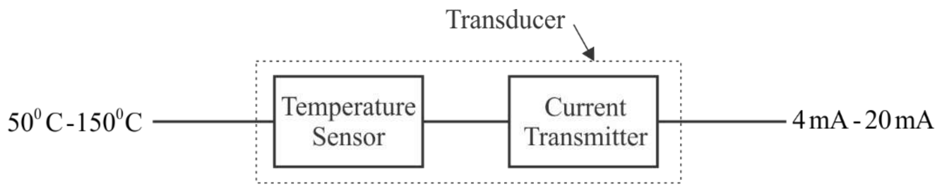
(C) $4\pi \in \left(\dfrac{y^2}{y-x}\right)$

(D) $4\pi \in \left(\dfrac{y^2-xy}{x}\right)$

Ans. (A)

Question 21

A linear transducer is calibrated for the ranges shown in the figure. The gain of the transducer is ______ $mA/^0 C$ (rounded off to two decimal places).



Ans. 0.15 to 0.17

Question 22 **[Signals & Systems]**

Consider a filter defined by the difference equation

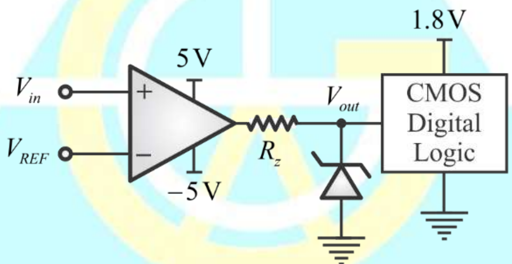
$y[n]-0.5y[n-2]=ax[n-4]$ where $x[n]$ and $y[n]$ represent the input and output, respectively. If the magnitude response of the filter at $\omega=\frac{\pi}{2}$ is $\left|H\left(\frac{\pi}{2}\right)\right|=0.5$, the value of a is _______.

(rounded off to two decimal places).

Ans. 0.70 to 0.80

Question 23 **[Analog Electronics]**

Consider the circuit shown in the figure.



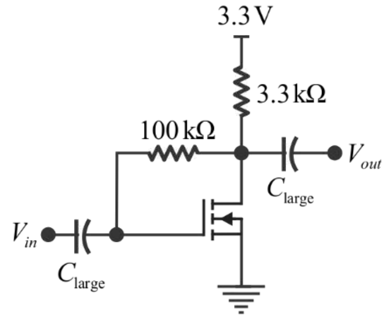
The CMOS digital logic circuit has infinite input impedance. Assume the opamp is ideal. A 1.8 V Zener diode with a minimum Zener current of 2 mA is used. The corresponding maximum value of resistance R_Z is ________ kΩ.

(rounded off to one decimal place).

Ans. 1.6 to 1.6

Question 24 **[Analog Electronics]**

Figure shows an amplifier using an NMOS transistor. Assume that the transistor is in saturation with device parameters, $\mu_n C_{ox}=250\,\mu A/V^2$, threshold voltage $V_T=0.65\,V$ and $W/L=4$. Ignore the channel length modulation effect. The drain current of the transistor at the operating point is _______ μA. (rounded off to nearest integer).



Ans. **498 to 502**

Question 25 **[Signals & Systems]**

The number of complex multiplications required for computing a 16-point DFT using the decimation-in-time radix-2 FFT is ________ (in integer).

Ans. **32 to 32**

Question 26

A 3×3 matrix P with all real elements has eigenvalues $\frac{1}{4}, 1,$ and -2. The value of $|P^{-1}|$ is _______ (rounded off to nearest integer).

Ans. **-2 to -2**

Sol. **Given :** A 3×3 matrix P with all real elements has eigenvalues $\frac{1}{4}, 1$ and -2.

As we know that :

$$PP^{-1} = I \text{ (Identity matrix)}$$

Taking determinant both side. We get,

$$|PP^{-1}| = |P||P^{-1}| = 1$$

$$|P^{-1}| = \frac{1}{|P|}$$

Since, the product of Eigen values of a matrix P is equal to the determinant of matrix P.

$|P| =$ Product of Eigen values

$$|P| = \left(\frac{1}{4}\right)(1)(-1) = \left(\frac{-1}{2}\right)$$

Hence, $$|P^{-1}| = \frac{1}{|P|} = \frac{1}{(-1/2)} = -2$$

$$|P^{-1}| = -2$$

Hence, the value of $|P^{-1}|$ is – 2

Question 27 **[Signals & Systems]**

The Nyquist sampling frequency for $x(t)=10\sin^2(200\pi t)$ is _________ Hz (rounded off to nearest integer).

Ans. **400 to 400**

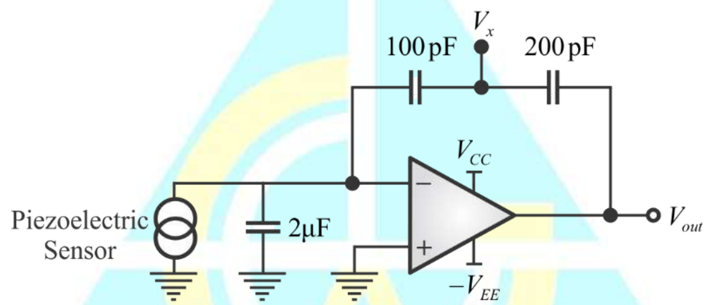
Question 28

The resistance of a 20 $k\Omega$ resistor is measured six consecutive times using an LCR meter. The first five readings are 19 $k\Omega$, 18 $k\Omega$, 23 $k\Omega$, 21 $k\Omega$ and 17 $k\Omega$. If the mean of the measurements and the true value are equal, the last reading is _________ $k\Omega$ (rounded off to nearest integer).

Ans. **22 to 22**

Question 29

Consider the readout circuit of a piezoelectric sensor shown in the figure.



When the piezoelectric sensor generates a charge q_p, the resulting change in voltage V_x is $-2V$. Then the corresponding change in the voltage V_{out} is _________V (rounded off to nearest integer).

Note : Assume all components are ideal.

Ans. **-3 to -3**

Question 30 **[Network Theory]**

The voltage applied and the current drawn by a circuit are

$v(t)=95+200\cos(120\pi t)+90\cos(360\pi t-60^0)V$

$i(t)=4\cos(120\pi t-60^0)+1.5\cos(240\pi t-75^0)A$

The average power absorbed by the circuit is _________ W (rounded off to nearest integer).

Ans. **200 to 200**

Sol. **Given :**

(i) Voltage $v(t)=95+200\cos(120\pi t)+90\cos(360\pi t-60^0)V$

(ii) Current $i(t)=4\cos(120\pi t-60^0)+1.5\cos(240\pi t-75^0)A$

Average power absorbed by the circuit.

$$P=V_{rms}I_{rms}\cos\phi$$

where $\phi=$Angle between V.

$$P=\frac{200}{\sqrt{2}}\times\frac{4}{\sqrt{2}}\cos\left(0^0-\left(-60^0\right)\right)$$

$$P=\frac{200}{\sqrt{2}}\times\frac{4}{\sqrt{2}}\cos\left(-60^0\right)=\frac{200\times 4}{2}\times\frac{1}{2}$$

$$P=200W$$

Hence the average power consumed by the circuit is 200 W.

Key point :

When different harmonics are present in a signal, the average power can be calculated by considering the power contribution from each harmonic component and then summing them up.

$\Rightarrow$ If any harmonic components are absent in the voltage or current expression, then there is no average power due to those absent components.

Question 31 **[Network Theory]**

The current $i(t)$ drawn by a circuit is given as

$$i(t)=4+30\cos(t)-20\sin(t)+15\cos(3t)-10\sin(3t)\,A$$

The root-mean-square value of $i(t)$ is __________ A (rounded off to one decimal place).

Ans. **27.0 to 30.0**

Sol. **Given :** The current $i(t)$ drawn by a circuit is

$$i(t)=4+30\cos(t)-20\sin(t)+15\cos(3t)-10\sin(3t)\,A$$

The RMS value of current $i(t)$ is given by.

$$I_{rms}=\sqrt{(4)^2+\left(\frac{30}{\sqrt{2}}\right)^2+\left(\frac{-20}{\sqrt{2}}\right)^2+\left(\frac{15}{\sqrt{2}}\right)^2+\left(\frac{-10}{\sqrt{2}}\right)^2}$$

$$=\sqrt{16+450+200+112.5+50}$$

$$=\sqrt{828.5}$$

$$I_{rms}=28.784\,A$$

Hence the correct answer is 29A.

Key point :

(1) Two harmonic of different frequency are always orthogonal.

$$x_1(t)=\sin\left(n\omega_0 t+\phi_1\right)$$

$$x_2(t)=\sin\left(m\omega_0 t+\phi_2\right)$$

Where, $n\neq m$

Now we will integrate then we get

$$\int_0^T \sin(n\omega_0 t+\phi_1)\sin(m\omega_0 t+\phi_2)\,dt = 0$$

(2) DC value and sine/cosine functions are orthogonal.

$$\int_0^T A\sin(n\omega_0 t+\phi)dt = 0$$

$$\int_0^T A\cos(n\omega_0 t+\phi)dt = 0$$

(3) Sine and cosine functions with same phase (ϕ) and same frequency (ω) are always orthogonal.

$$\int_0^T \sin(n\omega_0 t+\phi)\cos(n\omega_0 t+\phi)\,dt = 0$$

(4) Sine and cosine functions with same frequency but different phase angle. Then,

- $x(t) = A_1\cos(\omega_0 t+\phi_1) + A_2\cos(\omega_0 t+\phi_2)$

Or

- $x(t) = A_1\sin(\omega_0 t+\phi_1) + A_2\sin(\omega_0 t+\phi_2)$

$$A_0 = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos(\phi_1-\phi_2)}$$

Example :

$$x_1(t) = 2\sin 3t + 3\sin\left(3t+\frac{\pi}{3}+\frac{\pi}{2}\right)$$

$$R.M.S. = A_0 = \sqrt{4+9+12\cos\left(0-\frac{5\pi}{6}\right)}$$

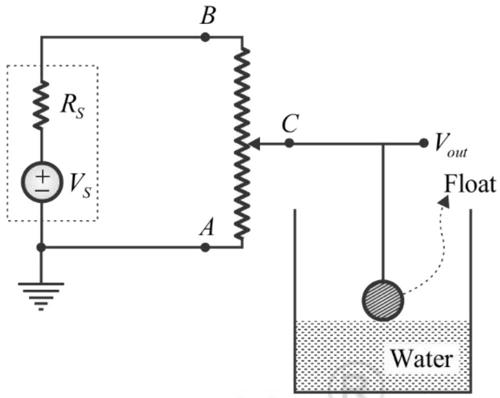
$$A_0 = 2.607$$

Now, resultant signal (or) function.

$$x(t) = 2.607\sin\left(3t-\frac{5\pi}{6}\right)$$

Question 32

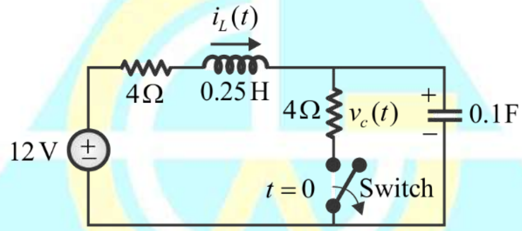
A linear potentiometer $(0-10\,\text{k}\Omega)$ is used to measure the water level as shown in the figure. The resistance between A and C varies linearly from 0 to $10\,k\Omega$ for a change in water level from 0 to 20 cm. The sensor is excited using a DC voltage source, $V_S = 10\,\text{V}$ with an internal resistance, $R_S = 200\,\Omega$. If $V_{out} = 5\,\text{V}$, the water level is _______ cm (rounded off to one decimal place).



Ans. 10.1 to 10.3

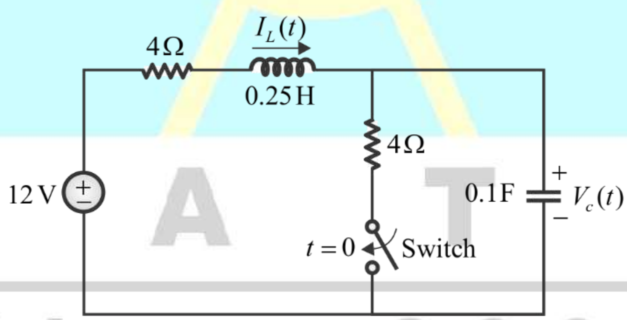
Question 33 **[Network Theory]**

The switch in the following figure has been closed for a long time $(t<0)$. It is opened at $t=0$ seconds. The value of dv_c/dt at $t=0^+$ is ________ V/s (rounded off to nearest integer).



Ans. 15 to 15

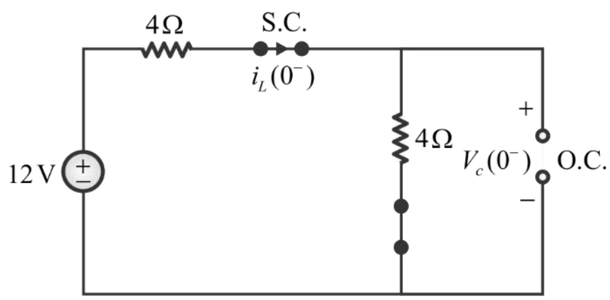
Sol. Given circuit is shown below.



(i) At $t=0^-/t<0/$ steady state :

Initially switch was closed.

In steady state, capacitor behaves as an open circuit and inductor behaves as an short circuit.



Applying KVL in above figure.

$$(4+4)i_L(O^-)=12$$

$$i_L(O^-)=\frac{12}{8}=1.5\text{ amp}$$

$$V_C(O^-)=4\times i_L(0^-)=4\times 1.5$$

$$V_C(O^-)=6\text{ Volt}$$

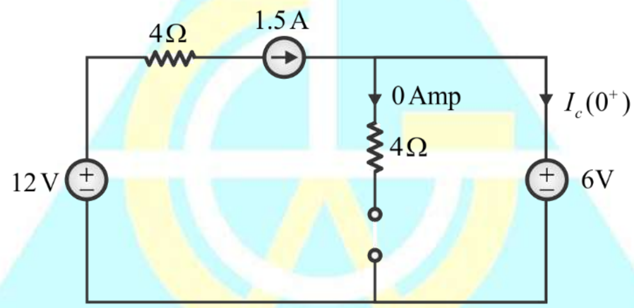
(ii) At $t=0^+$

- Inductor is replaced by a current source with initial value.

$$i_L(0^-)=i_L(0^+)=1.5\text{amp}$$

- Capacitor is replaced by a voltage source with initial value.

$$V_C(0^-)=V_C(O^+)=6\,volt$$



$$I_C(O^+)=1.5\text{ amp}$$

Current through capacitor is given by.

$$I_C(t)=c\frac{d}{dt}V_c(t)$$

At $t=0^+$

$$I_C(0^+)=c\frac{d}{dt}\left[V_c(0^+)\right]$$

$$\frac{dV_c(0^+)}{dt}=\frac{I_c(0^+)}{C}=\frac{1.5}{0.1}=15\text{ V/S}$$

Hence the correct answer is 15.

Question 34

Consider a system given by the following first order differential equation :

$$\frac{dy}{dt} = y + 2t - t^2$$

where, $y(0) = 1$ and $0 \le t < \infty$. Using a step size $h = 0.1$ for the improved Euler method, the value of $y(t)$ at $t = 0.1$ is ________(rounded off to two decimal places).

Ans. 1.10 to 1.12

Sol. Given :

(i) $\frac{dy}{dt} = y + 2t - t^2$ ...(i)

(ii) Step - size $h = 0.1$

(iii)Initial condition

$y(0) = 1$ i.e. $y_0 = 1, t_0 = 0$

From equation (i)

$$\frac{dy}{dt} = y + 2t - t^2 = f(t, y) \quad ...(ii)$$

$$f(t_0, y_0) = y_0 + 2t_0 - t_0^2$$

$$f(t_0, y_0) = 1 + 2(0) - 0$$

$$f(t_0, y_0) = 1$$

From Euler's improved method.

$$y_{n+1}^C = y_n + \frac{h}{2}[f(t_n, y_n) + f(t_{n+1}, y_{n+1}^p)] \quad ...(iii)$$

Where $y_{n+1}^p = y_n + hf(t_n, y_n)$ ...(iv)

and $t_{n+1} = t_n + h$ ...(v)

At first step $n = 0$ and $h = 0.1$

From equation (iv) ....(iv)

$$y_1^p = y_0 + hf(t_0, y_0)$$

$$y_1^p = 1 + 0.1[1]$$

$$y_1^p = 1.1$$

From equation (v)

$$t_{n+1} = t_n + h$$

$$t_1 = t_0 + h$$

$$t_1 = 0 + 0.1$$

$$t_1 = 0.1$$

From equation (ii)

$$f(t_1, y_1^p) = y_1^p + 2t_1 - t_1^2$$

$$= 1.1 + 2(0.1) - (0.1)^2$$

$$= 1.1 + 0.2 - 0.01$$

$$f(t_1, y_1^p) = 1.29$$

From equation (iii)

$$y_1^C = y_0 + \frac{h}{2}[f(t_0, y_0) + f(t_1, y_1^p)]$$

$$y_1^C = 1 + \frac{0.1}{2}[1 + 1.29]$$

$$y_1^C = 1 + 0.1145$$

$$y_1^C = 1.1145$$

$$y_1^C = 1.11$$

Hence, the value of $y(t)$ at $t = 0.1$ is 1.11

Q.36 to Q.65 Carry TWO Marks Each

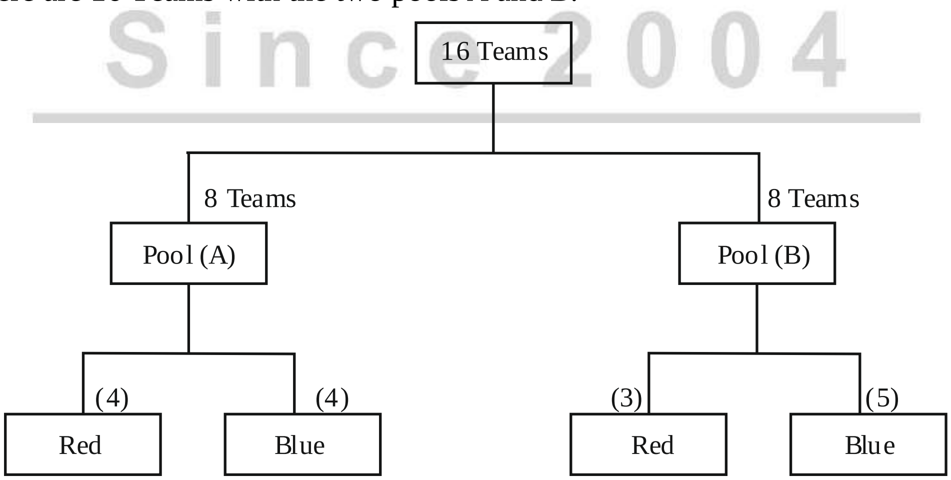
Question 35

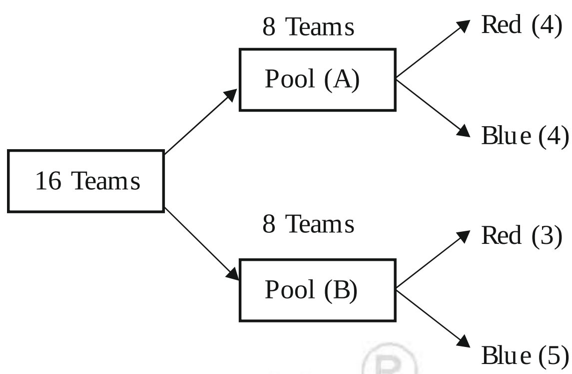
Indian Premier League has divided the sixteen cricket teams into two equal pools : Pool-A and Pool-B. Four teams of Pool-A have blue logo jerseys while the rest four have red log jerseys. Five teams of Pool-B have blue logo jerseys while the rest three have red logo jerseys.

If one team from each pool reaches the final, the probability that one team has a blue logo jersey and another has a red logo jersey is ________ (rounded off to one decimal place).

Ans. **0.5 to 0.5**

Sol. **Given :** There are 16 Teams with the two pools A and B.





Possible Cases $= P$ (Team A – Red) and P (Team B – Blue)

OR

P (Team A – Blue) and P (Team B – Red)

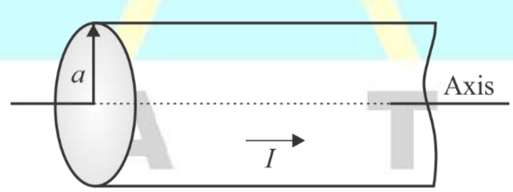
$$=\left[\left(\frac{4}{8}\right)\times\left(\frac{5}{8}\right)\right]+\left[\frac{4}{8}\times\frac{3}{8}\right]$$

$$=\frac{20}{64}+\frac{12}{64}=\frac{32}{64}=\frac{1}{2}=0.5$$

Hence, the probability that one team has a blue logo jersey and another has a read logo jersey is 0.5

Question 36

A wire of circular cross section with radius a is shown in the figure. The current density is given by $J = ks^2$, where k is a constant, s is the radial distance from the axis and $0 \le s \le a$. The total current I in the wire is



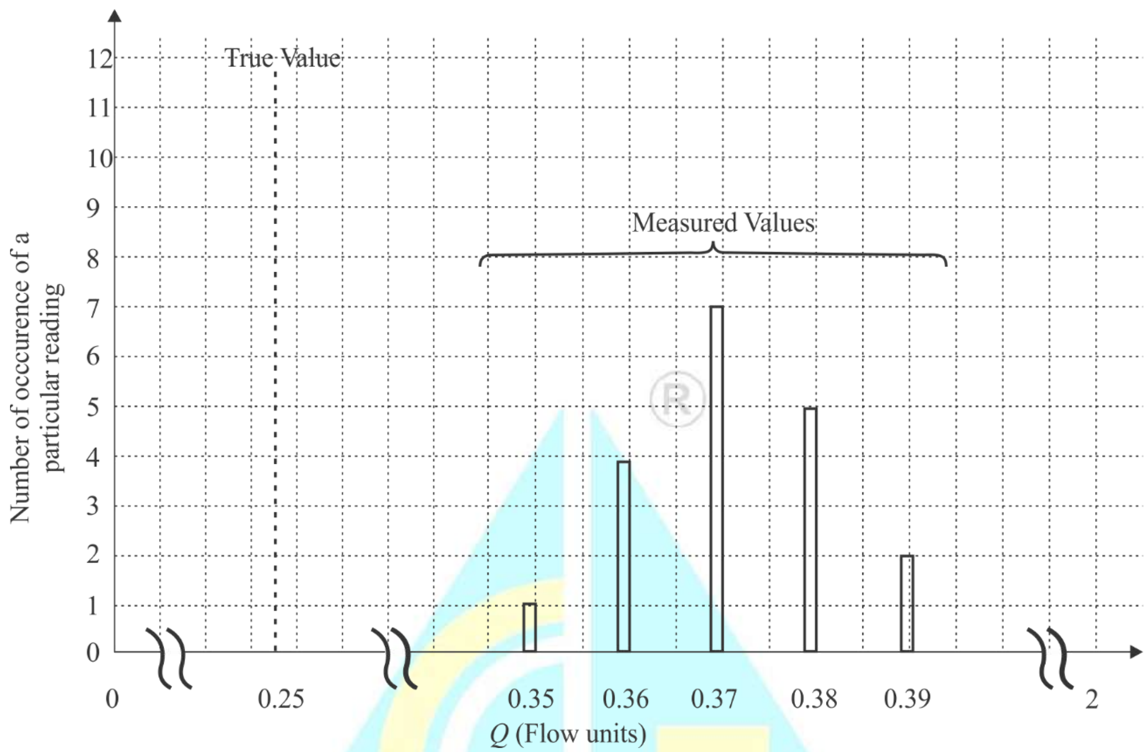
(A) $\dfrac{\pi k a^4}{2}$ (B) $\dfrac{2\pi k a^3}{3}$

(C) $\dfrac{\pi k a^3}{2}$ (D) $\dfrac{\pi k a^4}{4}$

Ans. (A)

Question 37

The measured values from a flow instrument, whose range is between 0 and 2 flow units, are shown in the histogram. The systematic error (bias) and the maximum error (in flow units), respectively are



(A) 0.12 and 0.14
(B) 0.01 and 0.10
(C) 0.10 and 0.14
(D) 0.04 and 0.12

Ans. (A)

Question 38 **[Signals & Systems]**

Consider a discrete-time sequence

$$x[n] = \begin{cases} (0.2)^n, & 0 \le n \le 7 \\ 0, & \text{otherwise} \end{cases}$$

The region of convergence of $X(z)$, the $z-$transform of $x[n]$, consists of

(A) All values of z except $z = 0.2$
(B) All values of z
(C) All values of z except $z = 0$
(D) All values of z except $z = \infty$

Ans. (C)

Sol. Given :

$$x(n) = \begin{cases} (0.2)^n & 0 \le n \le 7 \\ 0 & \text{otherwise} \end{cases}$$

$z-$Transform of discrete time signal $x[n]$ is given by

$$x(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

where, $z = re^{jw}$

$$x(z)=\sum_{n=0}^{7}(0.2)^n z^{-n}$$

$$x(z)=1+(0.2)z^{-1}+(0.2)^2 z^{-2}+(0.2)^3 z^{-3}+(0.2)^4 z^{-4}+(0.2)^5 z^{-5}+(0.2)^6 z^{-6}+(0.2)^7 z^{-7}$$

ROC : All values of z except $z=0$. Because at $z=0$. $z-$Transform of discrete time signal $x[n]$ will not exist.

Key point :

The value of $|z|$ for which $z-$Transform of signal $x[n]$ converges is known as region of convergence.

(1) For a rational z-transform $x(z)$. The ROC is bounded by poles or extends to infinity but ROC does not contain any poles.

(2) If $x[n]$ is of finite duration and absolutely summable, then the ROC is the entire z-plane. Excluding possibly $z=0$ (or) $z=\pm\infty$.

(3) For stability ROC includes unit circle in z-plane.

(4) For right sided signal. ROC will be outside circle in z-plane.

(5) For left sided signal, ROC will be inside circle in z-plane.

(6) For both sided signal ROC is a ring in z-plane.

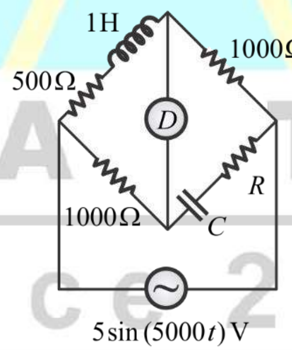
(7) ROC is depends on the nature of signal whether it is causal (or) anticausal.

(i) For causal signals. ROC will be outside circle in z-plane.

(ii) For anti causal signals ROC will be inside circle in z-plane.

Question 39

In the bridge circuit shown in the figure, under balanced condition, the values of R and C respectively, are

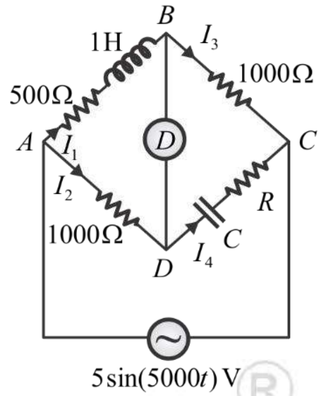


(A) 1.010 Ω and 19.802 μF (B) 9.901 Ω and 0.505 μF

(C) 19.802 Ω and 1.01 μF (D) 39.604 Ω and 2.02 μF

Ans. (C)

Sol. Given : Given bridge circuit is shown below



At balanced condition $(I_g = 0)$

- $V_{AB} = V_{AD}$

$$I_1 Z_1 \angle\theta_1 = I_2 Z_2 \angle\theta$$

$$\frac{I_2}{I_1} = \left|\frac{Z_1}{Z_2}\right| \angle\theta_1 - \theta_2 \qquad \text{......(i)}$$

- $V_{BC} = V_{DC}$

$$I_3 Z_3 \angle\theta_3 = I_4 Z_4 \angle\theta_4$$

$$I_1 Z_3 \angle\theta_3 = I_2 Z_4 \angle\theta_4$$

$$\frac{I_2}{I_1} = \left|\frac{Z_3}{Z_4}\right| \angle\theta_3 - \theta_4 \qquad \text{......(ii)}$$

From Equation (i) and (ii)

$$\left|\frac{Z_1}{Z_2}\right| \angle\theta_1 - \theta_2 = \left|\frac{Z_3}{Z_4}\right| \angle\theta_3 - \theta_4$$

(1) $\left|\frac{Z_1}{Z_2}\right| = \left|\frac{Z_3}{Z_4}\right|$

(2) $\angle\theta_1 - \theta_2 = \angle\theta_3 - \theta_4$

OR

$$\angle\theta_1 = \angle\theta_2 + \theta_3 - \theta_4$$

Now according to question:

(i) $Z_1 = 500 + j(5000)1$

$$|Z_1| = \sqrt{(500)^2 + (5000)^2}$$

$$|Z_1| = \sqrt{(500)^2[1+100]} = 500\sqrt{101}$$

$$|Z_1| = 5024.92\,\Omega$$

(ii) $Z_2 = 1000 + j0$

$$|Z_2| = 1000\ \Omega$$

(iii) $Z_3 = 1000\ \Omega$

(iv) $Z_4 = R_4 - jX_4$

Applying bridge balance condition i.e. product of opposite arm impedances are equal

$$Z_1 Z_4 = Z_2 Z_3$$

$$[500 + j(5000).1][R_4 - jX_4] = (1000)^2$$

$$[500 + j(5000)]\left[R_4 - \frac{-j}{\omega C_4}\right] = (1000)^2$$

$$500\,R_4 - \frac{j500}{\omega C_4} + jR_4(5000) + \frac{5000}{\omega C_4} = (1000)^2$$

$$\left[500\,\mathrm{R}_4 + \frac{5000}{\omega C_4}\right] + j\left[R_4(5000) - \frac{500}{\omega C_4}\right] = (1000)^2$$

Equating real and imaginary parts.

$$500\,R_4 + \frac{5000}{(5000)C_4} = (1000)^2$$

$$500\,R_4 + \frac{1}{C_4} = (1000)^2 \qquad \text{....(iii)}$$

$$R_4(5000) - \frac{500}{\omega C_4} = 0$$

$$R_4(5000) = \frac{500}{(5000)C_u}$$

$$R_4 = \frac{500}{(5000)(5000)C_u}$$

$$R_4 = \frac{1}{50000}C_u \qquad \text{....(iv)}$$

From eq. (iii) and (iv)

$$500\left(\frac{1}{50000\,C_u}\right) + \frac{1}{C_u} = (1000)^2$$

$$\frac{1}{100\,C_u} + \frac{1}{C_u} = (1000)^2 = 10^6$$

$$\frac{1}{C_u}\left[\frac{101}{100}\right] = 10^6$$

$$C_u = 1.01\mu\text{F}$$

From equation (iv)

$$R_u = \frac{1\times10^6}{50000\times1.01} = \frac{10^2}{5\times1.01}$$

$$R_u = 19.80\,\Omega$$

Hence, the correct option is (C)

Question 40 **[Signals & Systems]**

Laplace transform of a signal $x(t)$ is

$$X(s) = \frac{1}{s^2 + 13s + 42}$$

Let $u(t)$ be the unit step function. Choose the signal $x(t)$ from the following options if the region of convergence is $-7 < \text{Re}\{s\} < -6$.

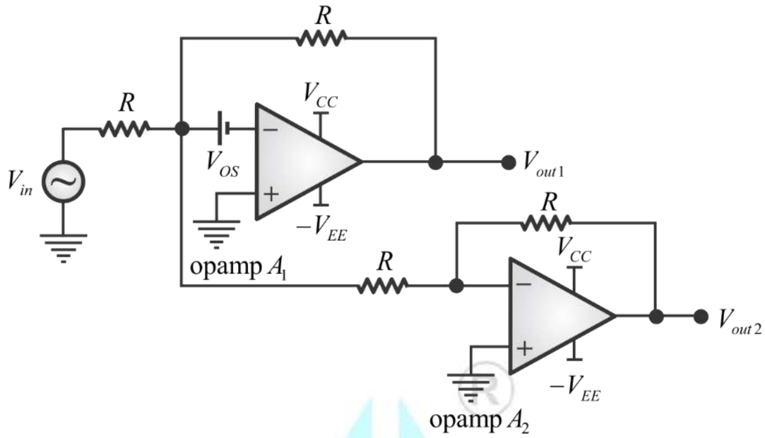
(A) $-e^{-6t}u(t) - e^{-7t}u(-t)$ (B) $-e^{-6t}u(-t) - e^{-7t}u(t)$

(C) $e^{-6t}u(t) - e^{-7t}u(-t)$ (D) $-e^{-6t}u(-t) - e^{-7t}u(-t)$

Ans. (B)

Question 41 **[Analog Electronics]**

In the figure shown, both the opamps A_1 and A_2 are ideal, except that the opamp A_1 has an offset voltage (V_{os}) of 1 mV. For $V_{in} = 0V$, the values of the output voltages V_{out1} and V_{out2}, respectively, are

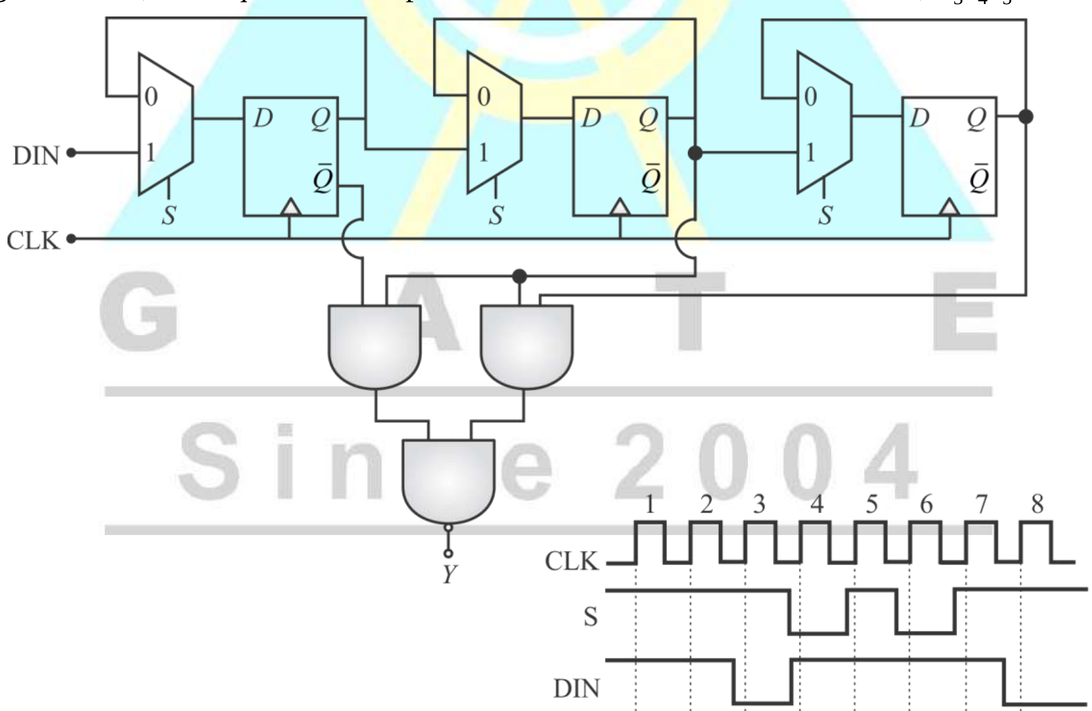


(A) 3mV and -1mV
(B) -1mV and 0mV
(C) 1mV and -1mV
(D) 2mV and 0mV

Ans. (A)

Question 42 **[Digital Electronics]**

In the figure shown, the positive edge triggered D flip-flops are initially reset to $Q = 0$. The logic gates and the multiplexers have no propagation delay. After reset, a train of clock pulses (CLK) are applied. The logic-states of the inputs DIN, S and the clock pulses are also shown in the figure. Assuming no timing violations, the sequence of output Y from the 3rd clock to the 5th clock, $Y_3Y_4Y_5$ is



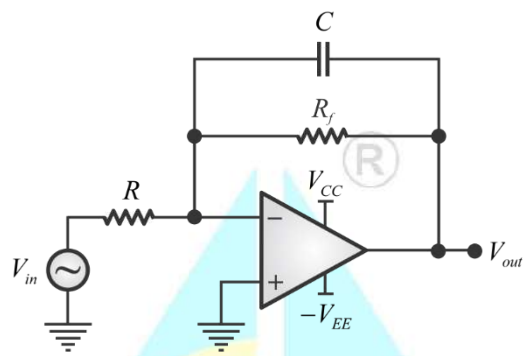
(A) 001
(B) 010
(C) 000
(D) 011

Ans. (A)

Question 43 **[Analog Electronics]**

In the figure shown, $R = 1k\Omega$ and $C = 0.1\mu F$. For a dc gain of -10, the 3 dB cut-off frequency (rounded off to one decimal place) is

Assume the op-amp is ideal.

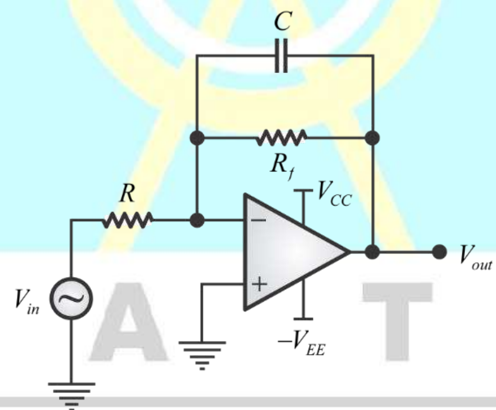


(A) 159.1 Hz (B) 1591.5 Hz

(C) 1750.7 Hz (D) 175.0 Hz

Ans. **(A)**

Sol. Given circuit with ideal op-amp is shown below.



(i) $R = 1\ K\Omega$ and $C = 0.1\ \mu F$

(ii) dc gain = –10

(iii) 3dB cut-off frequency = ?

Note : This is practical integrator circuit and practical integrator circuit is more stable compare to ideal integrator circuit.

Transfer function (T.F.) $= \dfrac{V_{out}}{V_{in}}$

$$\frac{V_{out}}{V_{in}} = \frac{-z_2}{z_1} = \frac{-\left[\dfrac{R_f \times \dfrac{1}{CS}}{R_f + \dfrac{1}{CS}}\right]}{R}$$

$$\frac{V_{out}}{V_{in}} = \frac{-R_f}{R\left(1 + SCR_f\right)}$$

$\therefore$ For dc gain $[s = 0]$

$$\frac{V_{out}}{V_{in}} = -\frac{R_f}{R} = \frac{-R_f}{1K}$$

$$\frac{-R_f}{1K} = -10 \text{ [Given]}$$

$$R_f = 10\,k\Omega$$

As we know that practical integrator circuit is also referred as first, order low pass fitter with upper 3-dB cut-frequency.

$$f_r = \frac{1}{2\pi R_f C_f} Hz$$

$$f_H = \frac{1}{2\pi \times 10 \times 10^3 \times 0.1 \times 10^{-6}}$$

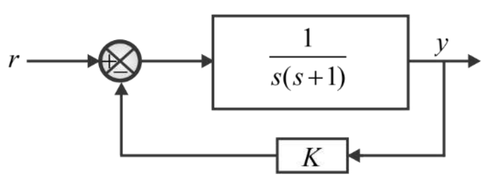
$$f_H = \frac{10^3}{2\pi} Hz$$

$$f_H = 159.155\,Hz$$

Hence the correct option is (A).

Question 44 **[Control System]**

Consider the feedback control system shown in the figure. The steady-state error $e_{ss} = \lim_{t \to \infty}\left(r(t) - y(t)\right)$ due to unit step reference $r(t)$ is



(A) $\frac{K-1}{K}$ (B) $\frac{1}{2}$

(C) 0 (D) $\frac{1-K}{K}$

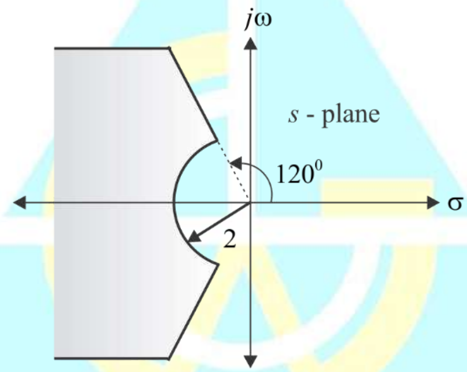
Ans. (A)

Question 45 **[Control System]**

The transfer function of a system is

$$G(s)=\frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$$

Choose the range of ξ and ω_n (in rad/s) from the following options such that the poles lie on the shaded region of the $s-$plane as shown in the figure.



(A) $\xi\geq\frac{1}{2}$ and $\omega_n\geq 2$ (B) $\xi\geq\frac{1}{4}$ and $\omega_n\geq 2$

(C) $\xi\geq\frac{1}{2}$ and $\omega_n\geq\sqrt{3}$ (D) $\xi\geq\frac{1}{4}$ and $\omega_n\geq\sqrt{3}$

Ans. (A)

Question 46

Let C be the closed curve in the $xy-$plane, traversed in the counterclockwise direction along the boundary of the rectangle with vertices at $(0,0),(2,0),(2,1),(0,1)$. The value of the line integral

$$\oint_C(-e^y dx+e^x dy)$$

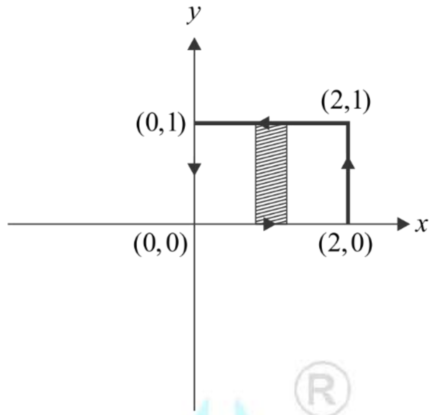
is **(2 Marks)**

(A) e^2+2e-3 (B) e^2-2e-3

(C) e^2+e-1 (D) e^2+e+1

Ans. (A)

Sol. **Given :** The rectangle with vertices at $(0,0),(2,0),(2,1),(0,1)$



(Encircles a surface)

The value of line integral $\oint_c \left(-e^{+y}dx + e^x dy\right)$

🕮 Key Points

Green theorem

$$\oint_c Mdx + Ndy = +\iint_s \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dxdy$$

Using Green theorem

$$\oint \left(-e^{y}dx + e^x dy\right) = +\iint_s \left(e^x + e^y\right) dxdy$$

$$\oint_c \left(-e^{+y}dx + e^x dy\right) = \int_0^2 \int_0^1 \left(e^x + e^y\right) dy\, dx$$

$$\oint_c \left(-e^{+y}dx + e^x dy\right) = \int_0^2 \left(e^x y + e^y\right)_0^1 dx$$

$$\oint_c \left(-e^{+y}dx + e^x dy\right) = \int_0^2 \left[\left(e^x + e\right) - \left(0+1\right)\right] dx$$

$$\oint_c \left(-e^{+y}dx + e^x dy\right) = \int_0^2 \left[e^x + \left(e-1\right)\right] dx$$

$$\oint_c \left(-e^{+y}dx + e^x dy\right) = \left[e^x + \left(e-1\right)x\right]_0^2$$

$$\oint_c \left(-e^{+y}dx + e^x dy\right) = \left[\left\{e^2 + \left(e-1\right)2\right\} - \left\{1\right\}\right]$$

$$\oint_c \left(-e^{+y}dx + e^x dy\right) = \left[e^2 + 2e - 2 - 1\right]$$

$$\oint_c \left(-e^{+y}dx + e^x dy\right) = \left[e^2 + 2e - 3\right]$$

Hence the correct option is (A).

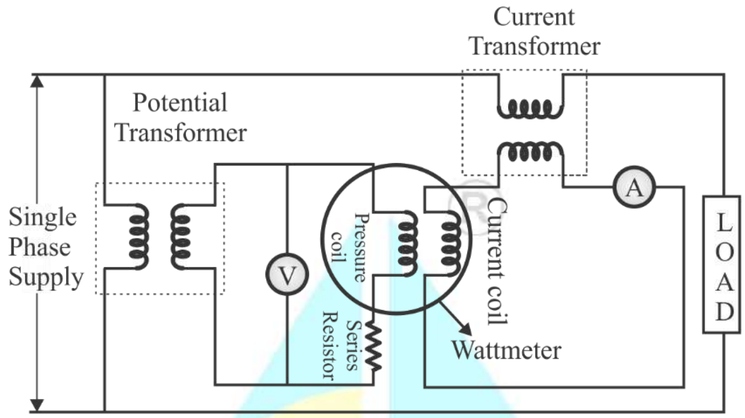
Question 47

In the figure shown, assume

- α is the phase angle between the load current and the load voltage
- β is the phase angle by which pressure coil current lags the pressure coil voltage of the wattmeter

- γ is the phase angle between currents in the pressure coil and the current coil of the wattmeter
- δ is the phase angle of the voltage transformer
- θ is the phase angle of the current transformer

When the load has a lagging phase angle of α, which one of the following options is correct ?



(A) $\alpha = -\gamma \pm \delta \pm \theta - \beta$ (B) $\alpha = -\gamma \pm \delta \pm \theta + \beta$

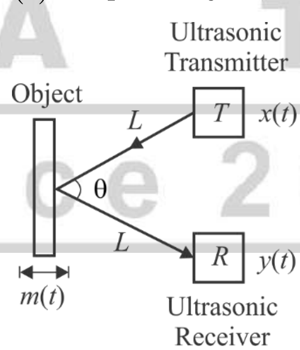
(C) $\alpha = \gamma \pm \delta \pm \theta + \beta$ (D) $\alpha = \gamma \pm \delta \pm \theta - \beta$

Ans. (C)

Question 48

Consider an ultrasonic measurement system shown in the figure. The ultrasonic transmitter (T) sends a continuous wave signal $x(t) = \cos(2\pi f_1 t)$ volts towards an object whose vibration is modeled as $m(t) = 0.5\sin(2\pi f_2 t)$ volts. Neglecting the phase shift due to any other effect, the received signal at the receiver (R) is $y(t) = \cos(2\pi f_1 t + \beta\cos(2\pi f_2 t))$ volts.

Assuming the frequency sensitivity factor as $500\,\text{Hz/volt}$, $f_1 = 40\,\text{kHz}$, $f_2 = 1\,\text{kHz}$, the modulation index (β) and the frequency deviation in $y(t)$, respectively, are



(A) 0.25 and $\pm 250\,\text{Hz}$ (B) 0.5 and $\pm 500\,\text{Hz}$

(C) 1 and $\pm 1000\,\text{Hz}$ (D) 0.75 and $\pm 1000\,\text{Hz}$

Ans. (A)

Question 49 **[MSQ]**

The complex functions $f(z)=u(x,y)+iv(x,y)$ and $\overline{f(z)}=u(x,y)-iv(x,y)$ are both analytic in a given domain. Choose the correct option(s) from the following. **(2 Marks)**

(A) $\frac{\partial u}{\partial x}=\frac{\partial v}{\partial y}=0$

(B) $\frac{\partial u}{\partial y}=-\frac{\partial v}{\partial x}\neq 0$

(C) $\frac{df(z)}{dz}=0$

(D) $\frac{df(z)}{dz}\neq 0$

Ans. **(A, C)**

Sol. **Given :** The complex function

$$f(z)=u(x,y)+iv(x,y)$$

and $\overline{f(z)}=u(x,y)-iv(x,y)$ are both analytic in a given domain.

Since $f(z)$ is analytic it satisfies Cauchy - Riemann equation

i.e. $$\frac{\partial u}{\partial x}=\frac{\partial v}{\partial y} \text{ and } \frac{\partial u}{\partial y}=\frac{-\partial v}{\partial x}$$

Since $\overline{f(z)}$ is analytic it satisfies Cauchy - Riemann equation

$$\overline{f(z)}=u(x,y)+\{-v(x,y)\}$$

i.e. $$\frac{\partial u}{\partial x}=\frac{-\partial v}{\partial y} \text{ and } \frac{\partial u}{\partial y}=\frac{\partial v}{\partial x}$$

$$\frac{\partial v}{\partial y}=\frac{-\partial v}{\partial y} \text{ and } \frac{\partial v}{\partial x}=\frac{-\partial v}{\partial x}$$

$$\frac{\partial v}{\partial y}=0 \text{ and } \frac{\partial v}{\partial x}=0$$

Similarly, $$\frac{\partial u}{\partial x}=0 \text{ and } \frac{\partial u}{\partial y}=0$$

Hence, u = constant, v = constant

Therefore, $f(z)=u+iv=\text{constant}$

Hence, $$\frac{df(z)}{dz}=0$$

Hence the correct options are (A) and (C).

Question 50

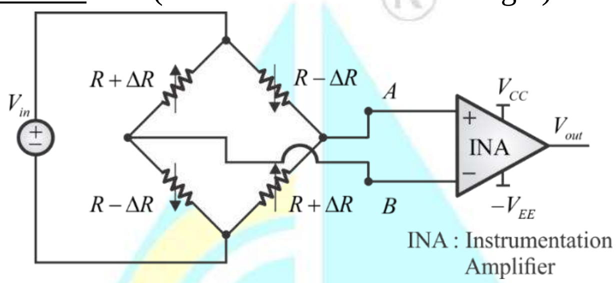
The readings recorded from a 20-psig pressure gauge are given in the Table. The regression line obtained for the data is $y = 0.04\,x + 10.32$. The regression coefficient of determination, $R^2 =$ ______ (rounded off to three decimal places).

x	1	2	3	4	5
y (psig)	10.3	10.5	10.4	10.5	10.5

Ans. 0.450 to 0.550

Question 51

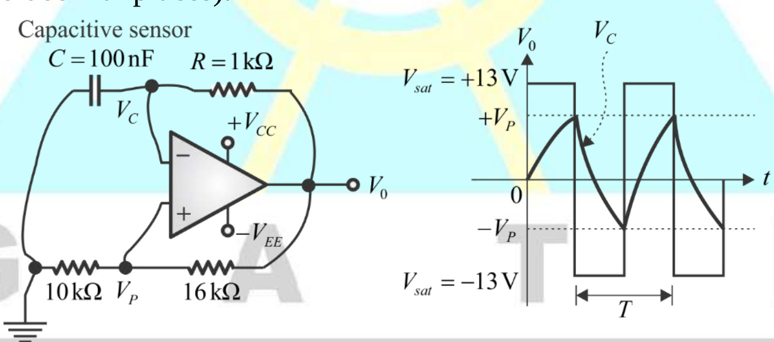
In the figure shown, $R = 4.5\ \text{k}\Omega$, $\Delta R = 1.5\ \text{k}\Omega$, and INA is assumed to be ideal. The equivalent resistance between A and B is ________ $\text{k}\Omega$ (rounded off to nearest integer).



Ans. 4 to 4

Question 52

Consider the capacitive sensor circuit and its output voltage shown in the figure. The circuit is switched ON at $t = 0$. Assuming the opamp to be ideal, the frequency of the output voltage V_0 is _______ kHz (rounded off to two decimal places).



Ans. 6.00 to 6.30

Question 53 **[Signals & Systems]**

The 4-point DFTs of two sequences $x[n]$ and $y[n]$ are $X[k] = [1, -j, 1, j]$ and $Y[k] = [1, 3j, 1, -3j]$, respectively. Assuming $z[n]$ represents the 4-point circular convolution of $x[n]$ and $y[n]$, the value of $z[0]$ is __________ (rounded off to nearest integer).

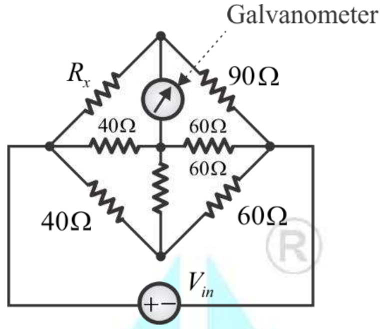
Note : The DFT of a N-point sequence $x[n]$ is defined as

$$X[k] = \sum_{n=0}^{N-1} x[n] e^{\frac{-j2\pi nk}{N}}$$

Ans. 2 to 2

Question 54

Consider the figure shown. For zero deflection in the galvanometer, the required value of resistor R_x is _______ Ω (rounded off to nearest integer).

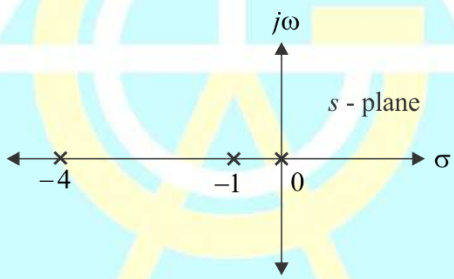


Ans. 58 to 62

Question 55 **[Control System]**

Consider a unity negative feedback system with its open-loop pole-zero map as shown in the figure. If the point $s = j\alpha, \alpha > 0$, lies on the root locus, the value of α is _______ (rounded off to nearest integer).

Note : The poles are marked with $\times$ in the figure.

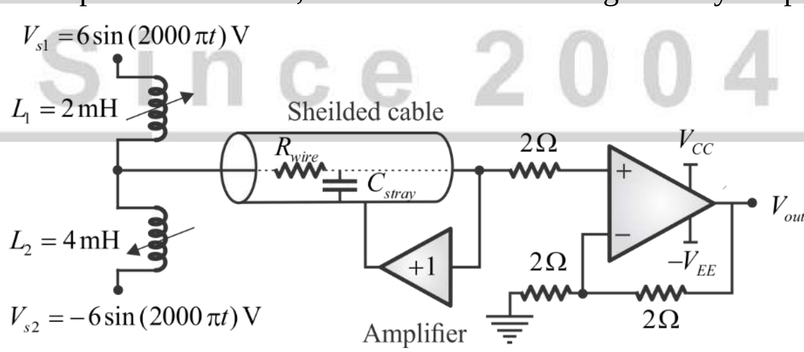


Ans. 2 to 2

Question 56

A shielded cable with $C_{stray} = 20\,pF$ and $R_{wire} = 10\,\Omega$ is used to connect the inductive sensors as shown in the figure. The RMS value of V_{out} is _______ V (rounded off to two decimal places).

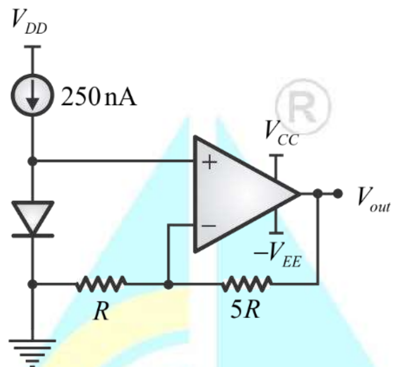
Note : Assume all components are ideal, and sensors are not magnetically coupled.



Ans. 2.81 to 2.85

Question 57 **[Analog Electronics]**

In the figure shown, the diode current is given by $I_D = I_S e^{\frac{\alpha V_D}{T}} . V_D$ is the diode voltage in volts, T is the absolute temperature in Kelvin, $\alpha = 1.16 \times 10^4 \, K/V$, and $I_S = 10^{-15} \, A$ is the saturation current. The dc current source, opamp and the resistors are ideal, and are assumed to be temperature independent. The change in the output voltage (V_{out}) per Kelvin change in temperature is ________ mV (rounded off to one decimal place).



Ans. 9.5 to 10.5

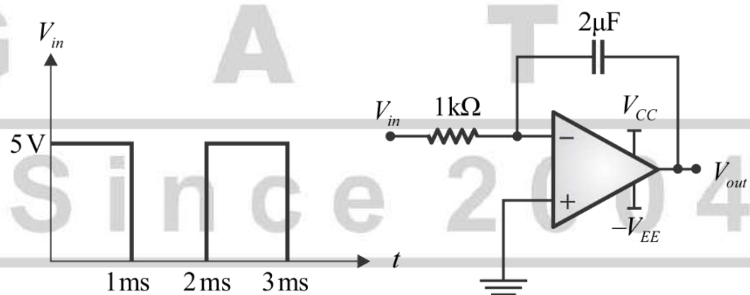
Question 58 **[Digital Electronics]**

An ADC has a full scale voltage of 1.4 V, resolution of 200 mV, and produces binary output data. The input signal of the ADC has a bandwidth of 500 MHz, and it samples the data at the Nyquist rate. The parallel data output is converted to a serial bit stream using a parallel-to-serial converter. The data rate at the output of the parallel-to-serial converter is _______ Gbps (rounded off to nearest integer).

Ans. 3 to 3

Question 59 **[Analog Electronics]**

In the circuit shown, assume the opamp is ideal and the initial charge on the capacitor is zero. The output voltage at time $t = 2\,ms$ is ________ V (rounded off to one decimal place).

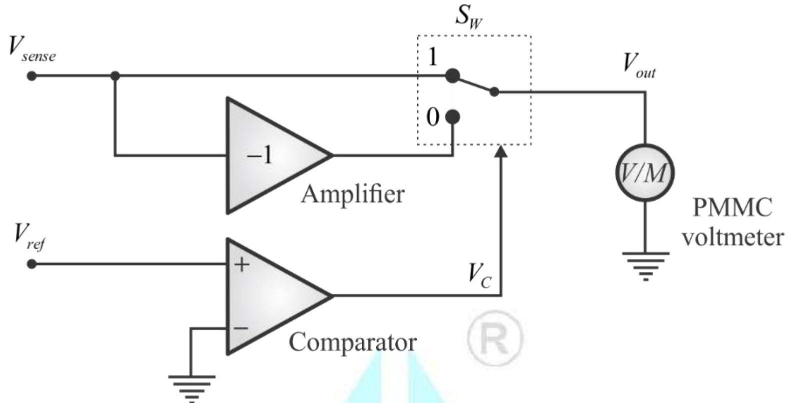


Ans. -2.6 to -2.4

Question 60 **[Analog Electronics]**

In the figure shown, S_W is a switch whose position changes from 1 to 0 when V_C change from logic HIGH to LOW and vice versa. The bandwidth of the permanent magnet moving coil (PMMC) type voltmeter is 1 Hz. If $V_{sense} = 2\sin(4000\,\pi t)$ V and $V_{ref} = 4\sin(2000\pi t)$ V, the voltmeter reading is _______ V (rounded off to nearest integer).

Note : Assume all components are ideal.



Ans. 0 to 0

Question 61

A 50 kVA transformer has an efficiency of 95% at full load and unity power factor. Assume the core losses are negligible. The efficiency of the transformer at 75% of the full load and 0.8 power factor is ________% (rounded off to one decimal place).

Ans. 95.2 to 95.4

Sol. Given :

(i) KVA rating S = 50 KVA

(ii) Full load efficiency. $\eta = 0.95$

(iii)Power factor, $\cos\phi$ = 1 (unity)

(iv)Assume core losses are negligible.

Efficiency at any load is given by .

$$\eta = \frac{P_{out}}{P_{in}} \times 100 = \frac{P_{out}}{P_{out} + \text{Losses}} \times 100$$

$$\eta = \frac{m\,\text{kVA}\,\cos\phi}{m\,\text{kVA}\,\cos\phi + P_i + m^2 P_{cufl}}$$

Where, m = Percentage of Loading

$\cos\phi$ = Power factor

Case 1 : At full load

$$m = 1,\ \cos\phi = 1$$

$$\eta = \frac{\text{kVA}}{\text{kAV} + P_i + P_{cufl}}$$

$$\eta = \frac{\text{kVA}}{\text{kAV} + P_{cufl}} \qquad \therefore \qquad [P_i \rightarrow \text{Core Losses are Negligible}]$$

$$0.95 = \frac{50 \times 10^3}{50 \times 10^3 + P_{cufl}}$$

$$50 \times 10^3 + P_{Cufl} = \frac{50 \times 10^3}{0.95}$$

$$P_{Cufl} = \left(\frac{1}{0.95} - 1 \right) 50 \times 10^3$$

$$P_{Cufl} = \frac{0.05}{0.95} \times 50 \times 10^3$$

$$P_{Cufl} = 2.6316\,\text{KW}$$

Case 2: At 75% of the full load

$$\cos\phi = 0.8$$

$$m = \frac{75}{100} = \frac{3}{4}$$

$$\eta = \frac{\left(\frac{3}{4}\right) \times 50 \times 10^3 \times 0.8}{\left(\frac{3}{4}\right) \times 50 \times 10^3 \times 0.8 + \left(\frac{3}{4}\right)^2 \times 2.6316 \times 10^3}$$

$$\eta = \frac{50 \times 0.8}{50 \times 0.8 + \left(\frac{3}{4}\right) \times 2.6316}$$

$$\eta = \frac{40}{40 + 1.9737} \times 100 = \frac{40}{41.9737} \times 100$$

$$\eta = 95.298\% \approx 95.3\%$$

Hence the correct answer is 95.3.

Question 62

A three-phase squirrel-cage induction motor has a starting torque of 100% of the full load torque and a maximum torque of 300% of the full load torque. Neglecting the stator impedance, the slip at the maximum torque is _______ % (rounded off to two decimal places).

Ans. **17.00 to 17.30**

Question 63 **[Network Theory]**

Two magnetically coupled coils, when connected in series-aiding configuration, have a total inductance of 500 mH. When connected in series-opposing configuration, the coils have a total inductance of 300 mH. If the self-inductance of both the coils are equal, then the coupling coefficient is ________ (rounded off to two decimal places).

Ans. **0.25 to 0.25**

Question 64

The solution of an ordinary differential equation $y'''+3y''+3y'+y=30e^{-t}$ is

$$y(t)=\left(c_0+c_1t-c_2t^2+c_3t^3\right)e^{-t}$$

Given $y(0)=3, y'(0)=-3$ and $y''(0)=-47$, the value of $\left(c_0+c_1+c_2+c_3\right)$ is _______ (rounded off to nearest integer).

Note : $y'''=d^3y/dt^3, y''=d^2y/dt^2, y'=dy/dt$ and c_0,c_1,c_2,c_3 are constants. **(2 Marks)**

Ans. **33 to 33**

Sol. **Given :** Differential equation

$$y'''+3y''+3y'+y=30e^{-t} \qquad \text{... (i)}$$

and its solution is

$$y(t)=\left(c_0+c_1t-c_2t^2+c_3t^3\right)e^{-t} \qquad \text{... (ii)}$$

Initial Conditions $y(0)=3, y'(0)=-3$ and $y''(0)=-47$

Taking Laplace transform of equation (i)

$$[s^3Y(s)-s^2y(0)-sy'(0)-y''(0)]+3[s^2Y(s)-sy(0)-y'(0)]$$

$$+3\left[sY(s)-y(0)\right]+Y(s)=\frac{30}{s+1} \qquad \left[\because L[e^{-at}]=\frac{1}{s+1}\right]$$

Using initial conditions

$$[s^3Y(s)-s^2(3)-s(-3)-(-47)]+3[s^2Y(s)-s(3)-(-3)]+3[sY(s)-3]+Y(s)=\frac{30}{s+1}$$

$$[s^3Y(s)-3s^2+3s+47]+3[s^2Y(s)-3s+3]+3[sY(s)-3]+Y(s)=\frac{30}{s+1}$$

$$Y(s)[s^3+3s^2+3s+1]+[-3s^2+3s+47-9s+9-9]=\frac{30}{s+1}$$

$$Y(s)[s+1]^3+[-3s^2-6s+47]=\frac{30}{s+1}$$

$$Y(s)[s+1]^3=\frac{30}{s+1}+3s^2+6s-47$$

$$Y(s)=\frac{30}{(s+1)^4}+\frac{3s^2+6s-47}{(s+1)^3}$$

$$Y(s)=\frac{30}{(s+1)^4}+\frac{3(s^2+2s+1)}{(s+1)^3}-\frac{50}{(s+1)^3}$$

$$Y(s)=\frac{30}{(s+1)^4}+\frac{3}{s+1}-\frac{50}{(s+1)^3}$$

$$Y(s)=\frac{30}{(s+1)^4}+\frac{3}{s+1}-\frac{50}{(s+1)^3} \qquad \left[L^{-1}\left[\frac{n!}{(s+a)^{n+1}}\right]=t^n e^{-at}u(t)\right]$$

Taking inverse Laplace transform

$$y(t)=\frac{30t^3e^{-t}}{3!}+3e^{-t}-\frac{50t^2e^{-t}}{2!}$$

$$y(t)=\frac{30t^3e^{-t}}{6}+3e^{-t}-\frac{50t^2e^{-t}}{2}$$

$$y(t)=5t^3e^{-t}+3e^{-t}-25t^2e^{-t}$$

$$y(t)=(3-25t^2+5t^3)e^{-t} \qquad \ldots \text{(iii)}$$

From equation (ii)

$$y(t)=\left(c_0+c_1t-c_2t^2+c_3t^3\right)e^{-t} \qquad \ldots \text{(ii)}$$

Comparing equation (ii) and (iii)

$c_0=3,$

$c_1=0,$

$c_2=25$

$c_3=5$

Value of $c_0+c_1+c_2+c_3=3+25+5=33$

Hence, the value of $(c_0+c_1+c_2+c_3)$ is 33.

Question 65

A random variable X has a probability density function

$$f_x(x)=\begin{cases} e^{-x}, & x\geq 0 \\ 0, & \text{otherwise}\end{cases}$$

The probability of $X>2$ is ________ (rounded off to three decimal places). **(2 Marks)**

Ans. 0.130 to 0.140

Sol. Given : $f_x(x)=\begin{bmatrix} e^{-x}, & x\geq 0 \\ 0, & \text{otherwise}\end{bmatrix}$

The probability is given by

$$P(-\infty\leq X\leq\infty)=\int_{-\infty}^{\infty} f_x(x)\,dx$$

Hence $P(X>2)=\int_2^{\infty} f_x(x)dx=\int_2^{\infty} e^{-x}dx$

$$=\left[\frac{e^{-x}}{(-1)}\right]_2^{\infty}=\frac{e^{-\infty}-e^{-2}}{(-1)}$$

$$=\frac{0-e^{-2}}{(-1)}=e^{-2}=\frac{1}{e^2}$$

$$P(X>2)=0.13533\approx 0.135$$

Hence, probability of $X>2$ is 0.135