

General Aptitude Part

Q.1 to Q.5 Carry One Mark Each

Question 1 Verbal Ability (Sentence Completion)

Rafi told Mary “I am thinking of watching a film this weekend”. The following reports the above statement in indirect speech.

Rafi told Mary that he _____ of watching a film this weekend.

- (A) thought (B) is thinking
(C) am thinking (D) was thinking

Ans. (D)

Sol. Rafi told Mary that he was thinking of watching a film this weekend.

Hence, the correct option is (D).

Question 2 Verbal ability (Idioms & Phrases)

Permit: _____ :: Enforce : Relax. (By word meaning)

- (A) Allow (B) Forbid
(C) License (D) Reinforce

Ans. (B)

Sol. Given :

Enforce is related to relax in a special manner, enforce and relax are opposite in meaning. In the same way of relationship opposite meaning of permit will be forbid.

Hence, the correct option is (B).

Question 3 Numerical Ability (Probability and Statistics)

Given a fair six - faced dice where the faces are labelled 1, 2, 3, 4, 5 and 6. What is the probability of getting a “1” on the first roll of dice and the “4” on the second roll?

- (A) $\frac{1}{36}$ (B) $\frac{1}{6}$
(C) $\frac{5}{6}$ (D) $\frac{1}{3}$

Ans. (A)

Sol. Probability of getting “1” on 1st roll = $\frac{1}{6}$

Probability of getting “4” on 2nd roll = $\frac{1}{6}$

Since, both events are independent

Probability of getting “1” on first roll and the “4” on the second roll = $\frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$

Hence, the correct option is (A).

Question 4 Verbal ability (case studies)

A recent survey shows that 65% of tobacco users were advised to stop consuming tobacco. The survey also shows that 3 out of 10 tobacco users attempted to stop using tobacco.

Based only on the interpretation of the above passage, which one of the following options can be logically inferred with certainty?

- (A) A majority of tobacco users who were advised to stop consuming tobacco made an attempt to do so.
- (B) A majority of tobacco users who were advised to stop consuming tobacco did not attempt to do so.
- (C) Approximately 30% of the tobacco users successfully stopped consuming tobacco.
- (D) Approximately 65% of tobacco users successfully stopped consuming tobacco.

Ans. (B)**Sol. Given :**

A recent survey shows that 65% of tobacco users were advised to stop consuming tobacco. The survey also shows that 3 out of 10 tobacco users attempted to stop using tobacco.

Option (C) cannot be inferred with certainty, as according to given information 3 out of 10 numerical tobacco users attempted to stop using tobacco, which is not a majority.

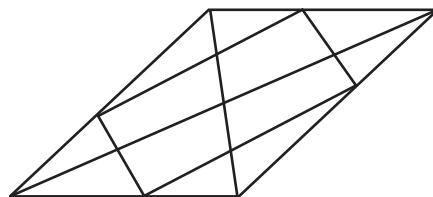
Option (A) and option (D) cannot be inferred with certainty, as there is no information about how much tobacco users successfully stop consuming tobacco.

Option (B) can be inferred with certainty, as 3 out 10 tobacco users attempted to stop using tobacco; which means 7 users did not attempted to do this, and 7 out of 10 is a majority.

Hence, the correct option is (B).

Question 5 Logical Reasoning (Numerical Computation)

How many triangles are present in the given figure?



(A) 12

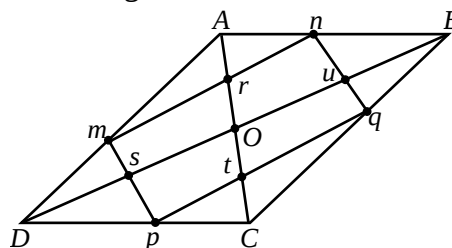
(B) 16

(C) 20

(D) 24

Ans. (C)**Sol. Given :**

Number of triangles are present in the figure,



In rectangle $ABCD$, number of triangle are 8.

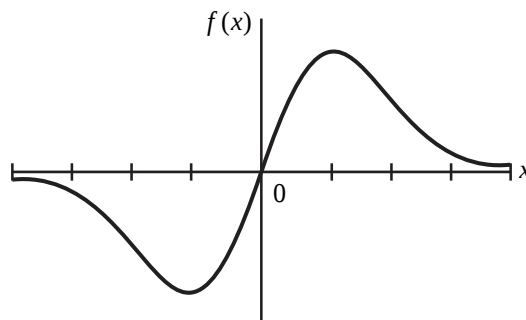
The triangles are, OAB , OBC , OCD , ODA , ADC , DCB , CBA , BAD .

Hence, the correct option is (C).

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Question 7 Logical Reasoning (numerical computation)

Which one of the following options represents the given graph?



(A) $f(x) = x^2 2^{-|x|}$

(B) $f(x) = x 2^{-|x|}$

(C) $f(x) = |x| 2^{-x}$

(D) $f(x) = x 2^{-x}$

Ans. (B)**Sol.** Only option (B) is odd function.

Hence, the correct option is (B).

Question 8 Verbal ability (case studies)

Which one of the options does NOT describe the passage below or follow from it?

We tend to think of cancer as a 'modern' illness because its metaphors are so modern. It is a disease of overproduction, of sudden growth, a growth that is unstoppable, tipped into the abyss of no control. Modern cell biology encourages us to imagine the cell as a molecular machine. Cancer is that machine unable to quench its initial command (to grow) and thus transform into an indestructible, self-propelled automaton.

[Adapted from *The Emperor of All Maladies* by Siddhartha Mukherjee]

- (A) It is a reflection of why cancer seems so modern to most of us.
- (B) It tells us that modern cell biology uses and promotes metaphors of machinery.
- (C) Modern cell biology encourages metaphors of machinery, and cancer is often imagined as a machine.
- (D) Modern cell biology never uses figurative language, such as metaphors, to describe or explain anything

Ans. (D)**Sol.** Given :

We tend to think of cancer as a 'modern' illness because its metaphors are so modern. It is a disease of overproduction, of sudden growth, a growth that is unstoppable, tipped into the abyss of no control. Modern cell biology encourages us to imagine the cell as a molecular machine. Cancer is that machine unable to quench its initial command (to grow) and thus transform into an indestructible, self-propelled automaton.

From the given information we can clearly see the modern cell biology uses figurative language, such as metaphors to describe modern illness cancer.

Hence, the correct option is (D).

Question 9 Numerical Ability (number system)

The digit in the unit's place of product $3^{999} \times 7^{1000}$ is ____.

- (A) 7 (B) 1
(C) 3 (D) 9

Ans. (A)

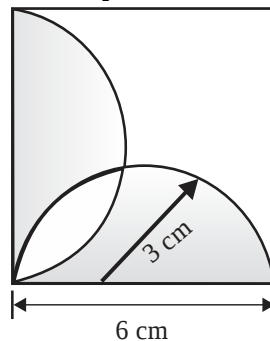
Sol. Given : $3^{999} \times 7^{1000} = (3^4)^{249} \times 3^3 \times (7^4)^{25} = 1 \times 27 \times 1$

Unit digit value = 7

Hence, the correct option is (A).

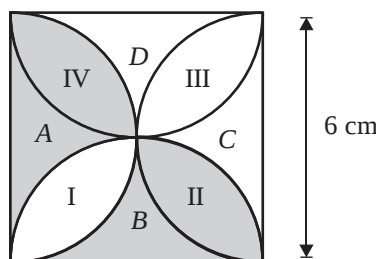
Question 10 Logical Reasoning (numerical computation)

A square with sides of length 6 cm is given. The boundary of the shaded region is defined by two semi-circles whose diameters are the sides of the square, as shown.



The area of the shaded region is _____ cm^2 .

- (A) 6π (B) 18
(C) 20 (D) 9π

Ans. (B)**Sol.**

$$\text{Area of region } (A + B + C + D) = 2[36 - (\pi \cdot 3^2)] = 72 - 18\pi$$

$$\text{Area of region } (I + II + III + IV) = 36 - [72 - 18\pi] = 18\pi - 36$$

$$\text{Area of region "I"} = (18\pi - 36)/4 = \frac{9\pi}{2} - 9$$

$$\text{Area of shared region} = \text{Area of } (A + B + I)$$

$$= \frac{\pi 3^3}{2} - I + \frac{\pi 3^2}{2} - I = \frac{9\pi}{2} + \frac{9\pi}{2} - 2\left(\frac{9\pi}{2} - 9\right)$$

$$= 9\pi - (9\pi - 18) = 18$$

Hence, the correct option is (B).

Technical Part

Q.11 to Q.35 Carry One Mark Each

Question 11 Mathematics (linear algebra)

For a given vector $W = [1, 2, 3]^T$, the vector normal to the plane defined by $W^T x = 1$ is

(A) $[-2, -2, 2]^T$

(B) $[3, 0, -1]^T$

(C) $[3, 2, 1]^T$

(D) $[1, 2, 3]^T$

Ans. (D)**Sol.** Given : $W = [1, 2, 3]$

$$W_x^T = 1$$

$$[1 \ 2 \ 3] \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 1$$

$$\phi : x + 2y + 3z = 1$$

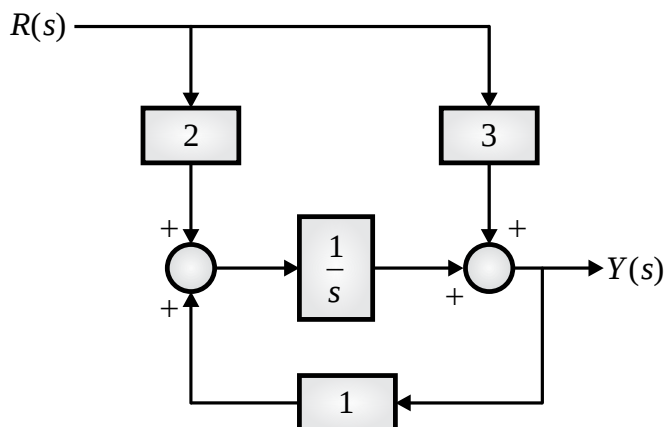
Normal vector or gradient of $\phi = \hat{i} + 2\hat{j} + 3\hat{k}$

Normal vector $= [1, 2, 3]^T$

Hence, the correct option is (D).

Question 12 Control system (block diagram & signal)

For the block diagram shown in the figure, the transfer function $\frac{Y(s)}{R(s)}$ is



(A) $\frac{2s+3}{s+1}$

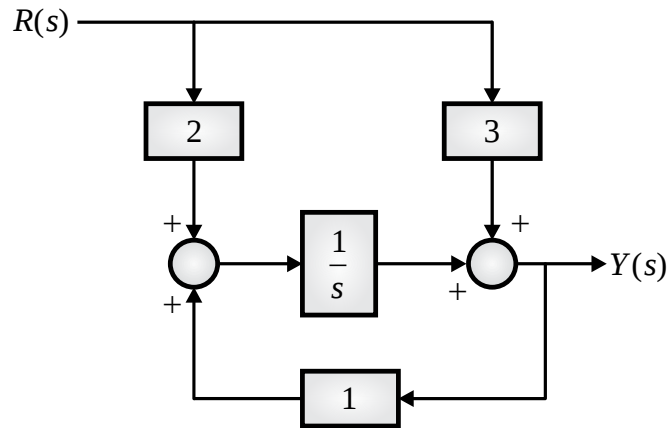
(B) $\frac{3s+2}{s-1}$

(C) $\frac{s+1}{3s+2}$

(D) $\frac{3s+2}{s+1}$

Ans. (B)

Sol. Given block diagram is,



By Mason's gain formula,

$$\frac{Y(s)}{R(s)} = \frac{P_1 \Delta_1 + P_2 \Delta_2}{\Delta}$$

$$P_1 = 3 \quad P_2 = \frac{2}{s}$$

$$\Delta_1 = 1 \quad \Delta_2 = 1$$

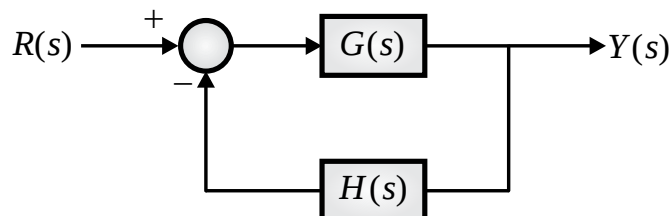
$$\Delta = 1 - \frac{1}{s}$$

$$\frac{Y(s)}{R(s)} = \frac{3 + \frac{2}{s}}{1 - \frac{1}{s}} = \frac{3s + 2}{s - 1}$$

Hence, the correct option is (B).

Question 13 Control system (Nyquist stability criteria)

In the Nyquist plot of the open-loop transfer function $G(s)H(s) = \frac{3s+5}{s-1}$ corresponding to the feedback loop shown in the figure, the infinite semicircular arc of the Nyquist contour in s-plane is mapped into a point at



(A) $G(s)H(s) = \infty$

(B) $G(s)H(s) = 0$

(C) $G(s)H(s) = 3$

(D) $G(s)H(s) = -5$

Ans. (C)

Sol. Given open loop transfer function, $G(s)H(s) = \frac{3s+5}{s-1}$

$$\Rightarrow G(s)H(s) = \frac{s\left(3 + \frac{5}{s}\right)}{s\left(1 - \frac{1}{s}\right)} = \frac{3 + \frac{5}{s}}{1 - \frac{1}{s}}$$

The infinite semicircular arc of the Nyquist contour in s-plane is mapped into a point at.

$$\Rightarrow G(s)H(s)\big|_{s=\infty} = \frac{3+0}{1-0} = 3$$

Hence, the correct option is (C).

Question 14 Control system (controller and compensator)

Consider a unity gain negative feedback system consisting of the plant $G(s)$ (given below) and a proportional integral controller. Let the proportional gain and integral gain be 3 and 1, respectively. For a unit step reference input, the final values of the controller output and the plant output, respectively, are

$$G(s) = \frac{1}{s-1}$$

(A) ∞, ∞

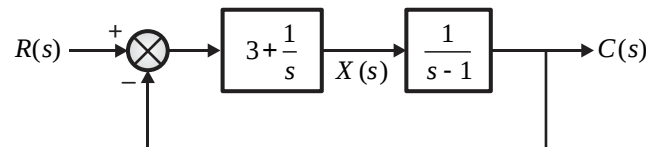
(B) 1, 0

(C) 1, -1

(D) -1, 1

Ans. (D)

Sol. Given system can be drawn as shown below,



Here,

$X(s) \rightarrow$ Controller output

$C(s) \rightarrow$ Plant output

$$C(s) = R(s) \frac{\left(3 + \frac{1}{s}\right) \left(\frac{1}{s-1}\right)}{1 + \left(3 + \frac{1}{s}\right) \frac{1}{s-1}} = R(s) \frac{(3s+1)}{s(s-1) + 3s+1} = \frac{(3s+1)}{s(s-1) + 3s+1} \times \frac{1}{s}$$

$$\therefore c(\infty) = \lim_{s \rightarrow 0} sC(s) = 1$$

$$X(s) = [R(s) - C(s)] \left(3 + \frac{1}{s}\right)$$

$$X(s) = [R(s) - C(s)] \left(\frac{3s+1}{s}\right)$$

$$x(\infty) = \lim_{s \rightarrow 0} sX(s) = \lim_{s \rightarrow 0} [R(s) - C(s)](3s+1)$$

$$\begin{aligned} x(\infty) &= \lim_{s \rightarrow 0} \left(\frac{1}{s} - \frac{1}{s} \frac{(3s+1)}{(s^2+2s+1)} \right) (3s+1) \\ &= \lim_{s \rightarrow 0} \frac{1}{s} \left[\frac{s^2+2s+1-3s-1}{s^2+2s+1} \right] (3s+1) \\ &= \lim_{s \rightarrow 0} \frac{1}{s} \left[\frac{s^2-s}{s^2+2s+1} \right] (3s+1) = \lim_{s \rightarrow 0} \frac{(s-1)(3s+1)}{s^2+2s+1} = -1 \end{aligned}$$

Hence, the correct option is (D).

Question 15 Electrical Machine (DC machine)

The following columns present various modes of induction machine operation and the ranges of slip :

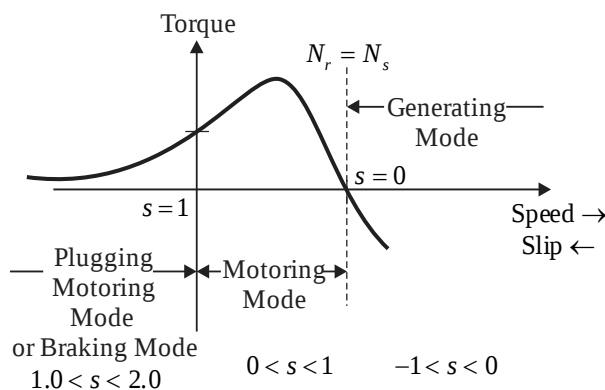
A	B
Mode of operation	Range of slip
(a) Running in generator mode	(p) From 0.0 to 1.0
(b) Running in motor mode	(q) From 1.0 to 2.0
(c) Plugging in motor mode	(r) From -1.0 to 0.0

The correct matching between the elements in column A with those of column B is

- | | |
|----------------------|----------------------|
| (A) a-r, b-p and c-q | (B) a-r, b-q and c-p |
| (C) a-p, b-r and c-q | (D) a-q, b-p and c-r |

Ans. (A)

Sol. Given :



Hence, the correct option is (A).

Question 16 Electrical machine (Single phase induction motor)

A 10 pole, 50 Hz, 240 V, single phase induction motor runs at 540 RPM while driving rated load. The frequency of induced rotor currents due to backward field is

- | | |
|------------|-----------|
| (A) 100 Hz | (B) 95 Hz |
| (C) 10 Hz | (D) 5 Hz |

Ans. (B)

Sol. Given : $P = 10$, $f = 50 \text{ Hz}$, $V_f = 240 \text{ V}$, $N_r = 540 \text{ rpm}$

The frequency of induced rotor current due to backward field is given by,

$$N_s = \frac{120f}{P} = \frac{120 \times 50}{10} = 600 \text{ rpm}$$

$$S_f = \frac{N_s - N_r}{N_s} = \frac{600 - 540}{600} = \frac{60}{600} = 0.1$$

As we know, backward field will rotate opposite to forward field so it will try to make the rotor in opposite direction so slip will be

$$S_b = \frac{N_s + N_r}{N_s} = \frac{600 + 540}{600} = \frac{1140}{600} = 1.9$$

As we know, frequency in the rotor is slip frequency.

So, frequency due to backward slip will be $S_b f_s = 1.9 \times 50 = 95 \text{ Hz}$

Hence, the correct option is (B).

Question 17 Signals & Systems (Laplace transform)

A continuous time system that is initially at rest is described by $\frac{dy(t)}{dt} + 3y(t) = 2x(t)$, where $x(t)$ is the input voltage and $y(t)$ is the output voltage. The impulse response of the system is

- (A) $3e^{-2t}$ (B) $\frac{1}{3}e^{-2t}u(t)$
(C) $2e^{-3t}u(t)$ (D) $2e^{-3t}$

Ans. (C)

Sol. Given : $\frac{dy(t)}{dt} + 3y(t) = 2x(t)$

Taking Laplace transform on both sides, we get

$$sY(s) + 3Y(s) = 2X(s)$$

$$\Rightarrow (s+3)Y(s) = 2X(s)$$

$$\Rightarrow \frac{Y(s)}{X(s)} = \frac{2}{s+3}$$

$$\Rightarrow H(s) = \frac{2}{s+3}$$

$\therefore$ Impulse response will be, $h(t) = L^{-1}(H(s)) = 2e^{-3t}u(t)$

Hence, the correct option is (C).

Question 18 Signals & Systems (Fourier transform)

The Fourier transform $X(\omega)$ of the signal $x(t)$ is given by

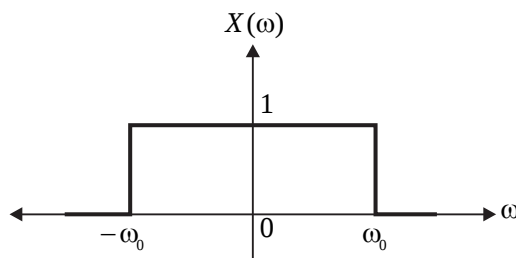
$$X(\omega) = \begin{cases} 1, & \text{for } |\omega| < \omega_0 \\ 0, & \text{for } |\omega| > \omega_0 \end{cases}$$

Which one of the following statements is true?

- (A) $x(t)$ tends to be an impulse as $\omega_0 \rightarrow \infty$ (B) $x(0)$ decreases as ω_0 increases
(C) At $t = \frac{\pi}{2\omega_0}$, $x(t) = \frac{-1}{\pi}$ (D) At $t = \frac{\pi}{2\omega_0}$, $x(t) = \frac{1}{\pi}$

Ans. (A)

Sol. Given : $X(\omega) = \begin{cases} 1, & \text{for } |\omega| < \omega_0 \\ 0, & \text{for } |\omega| > \omega_0 \end{cases}$



By taking inverse Fourier transform,

$$x(t) = \frac{\sin \omega_0 t}{\pi t}$$

$$x\left(\frac{\pi}{2\omega_0}\right) = \frac{2\omega_0}{\pi \times \pi} \sin \omega_0 \times \frac{\pi}{2\omega_0} = \frac{2\omega_0}{\pi^2} \sin \frac{\pi}{2} = \frac{2\omega_0}{\pi^2}$$

So, option (C) and (D) are wrong.

$$x(0) = \lim_{t \rightarrow 0} \frac{\sin \omega_0 t}{\pi t} = \lim_{t \rightarrow 0} \frac{\omega_0 \cos \omega_0 t}{\pi} = \frac{\omega_0}{\pi}$$

So, $x(0) \propto \omega_0 \Rightarrow$ Option (B) is wrong.

When $\omega_0 \rightarrow \infty$, $X(\omega)$ will be a D.C signal and inverse Fourier transform of a D.C signal will be impulse signal.

So, option (A) is correct.

Hence, the correct option is (A).

Question 19 Signals & Systems (Z transform)

The Z-transform of a discrete signal $x[n]$ is $X[z] = \frac{4z}{\left(z - \frac{1}{5}\right)\left(z - \frac{2}{3}\right)(z-3)}$ with ROC = R

Which one of the following statements is true?

- (A) Discrete time Fourier transform of $x[n]$ converges if R is $|z| > 3$
(B) Discrete time Fourier transform of $x[n]$ converges if R is $\frac{2}{3} < |z| < 3$
(C) Discrete time Fourier transform of $x[n]$ converges if R is such that $x[n]$ is a left-sided sequence.

(D) Discrete time Fourier transform of $x[n]$ converges if R is such that $x[n]$ is a right-sided sequence.

Ans. (B)

Sol. Given : $X(z) = \frac{4z}{\left(z - \frac{1}{5}\right)\left(z - \frac{2}{3}\right)(z - 3)}$

Poles of $X(z)$ are located at $z = \frac{1}{5}$, $z = \frac{2}{3}$ and $z = 3$.

For DTFT to converge, the ROC of Z-transform of $x(n)$ should contain unit circle.

If $x(n)$ is a right sided sequence then the ROC is $|z| > 3$ which does not include unit circle. So, option (D) and (A) are wrong.

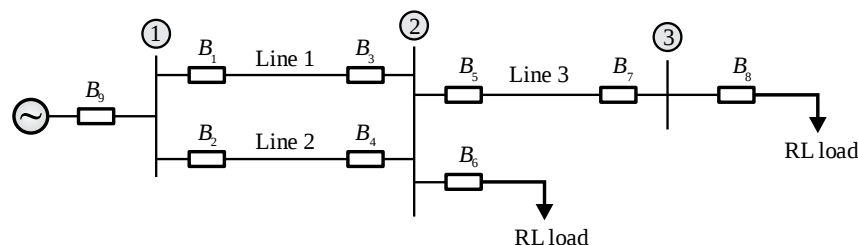
If R.O.C. is $\frac{2}{3} < |z| < 3$, the R.O.C. includes unit circle. So, option (B) is correct.

If $x(n)$ is a left sided then R.O.C will be $|z| < \frac{1}{5}$ which does not include unit circle. So, option (C) is wrong.

Hence, the correct option is (B).

Question 20 Power system (cable and insulator)

For the three-bus power system shown in the figure, the trip signals to the circuit breakers B_1 to B_9 are provided by overcurrent relays R_1 to R_9 , respectively, some of which have directional properties also. The necessary condition for the system to be protected for short circuit fault at any part of the system between bus 1 and the R-L loads with isolation of minimum portion of the network using minimum number of directional relays is

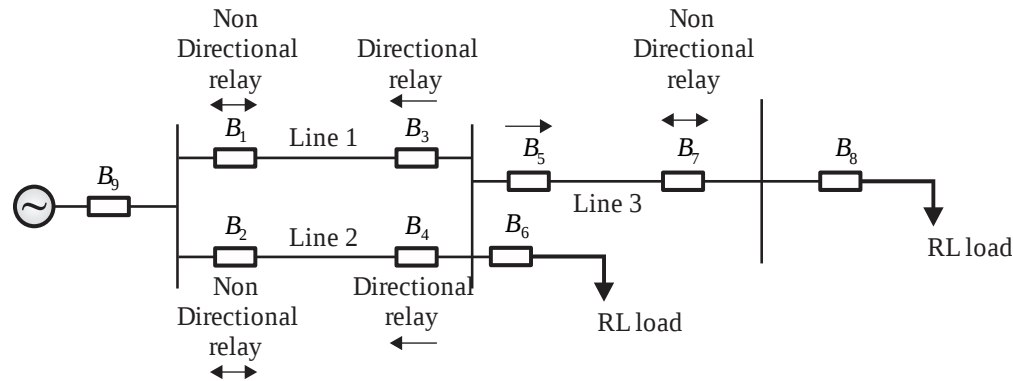


- (A) R_3 and R_4 are directional overcurrent relays blocking faults towards Bus 2.
- (B) R_3 and R_4 are directional overcurrent relays blocking faults towards Bus 2 and R_7 is directional overcurrent relay blocking faults towards bus 3.
- (C) R_3 and R_4 are directional overcurrent relays blocking faults towards Line 1 and Line 2, respectively, R_7 is directional overcurrent relay blocking faults towards Line 3 and R_5 is directional overcurrent relay blocking faults towards bus 2.

(D) R_3 and R_4 are directional overcurrent relays blocking faults towards Line 1 and Line 2, respectively.

Ans. (A)

Sol.



Hence, R_3 and R_4 are directional over current relays, which only operates when fault occurs in line-1 and line-2 respectively. But blocking faults towards bus-2.

Hence, the correct option is (A).

Question 21 Power system (per unit system)

The expressions of fuel cost of two thermal generating units as a function of the respective power generation P_{G_1} and P_{G_2} are given as

$$F_1(P_{G_1}) = 0.1aP_{G_1}^2 + 40P_{G_1} + 120 \text{ Rs/hr} \quad 0 \text{ MW} \leq P_{G_1} \leq 350 \text{ MW}$$

$$F_2(P_{G_2}) = 0.2P_{G_2}^2 + 30P_{G_2} + 100 \text{ Rs/hr} \quad 0 \text{ MW} \leq P_{G_2} \leq 300 \text{ MW}$$

Where a is a constant. For a given value of a , optimal dispatch requires the total load of 290 MW to be shared as $P_{G_1} = 175 \text{ MW}$ and $P_{G_2} = 115 \text{ MW}$. With the load remaining unchanged, the value of a is increased by 10% and optimal dispatch is carried out. The changes in P_{G_1} and the total cost of generation,

$F (= F_1 + F_2)$ in Rs/hr will be as follows

(A) P_{G_1} will decrease & F will increase

(B) Both P_{G_1} & F will increase

(C) P_{G_1} will increase & F will decrease

(D) Both P_{G_1} & F will decrease

Ans. (A)

Sol. Given :

$$F_1(P_{G_1}) = 0.1aP_{G_1}^2 + 40P_{G_1} + 120 \text{ Rs/hr}$$

$$F_2(P_{G_2}) = 0.2P_{G_2}^2 + 30P_{G_2} + 100 \text{ Rs/hr}$$

$$\text{Incremental cost, } I_{C_1} = \frac{dF_1}{dP_{G_1}} = 0.2aP_{G_1} + 40$$

$$I_{C_2} = \frac{dF_2}{dP_{G_2}} = 0.4P_{G_2} + 30$$

$$0.2a \times 175 + 40 = 0.4 \times 115 + 30$$

$$a = 1.028$$

$$\text{Now, } P'_1 + P'_2 = 175 + 115 = 290 \text{ mW}$$

$$a' = 1.1 \times 1.028 = 1.1308$$

$$F_1 = 0.1 \times 1.1308 P_1'^2 + 40 P_1' + 120$$

$$F_2 = 0.2 P_2'^2 + 30 P_2' + 100$$

$$I_{C_1} = 0.226 P_1' + 40$$

$$I_{C_2} = 0.4 P_2' + 30$$

For optimal load, $I_{C_1} = I_{C_2}$

$$0.226 P_1' + 40 = 0.4 P_2' + 30$$

$$0.226 P_1' - 0.4 P_2' = -10 \quad \dots(i)$$

$$P'_1 + P'_2 = 290$$

$$P'_1 = 169 \text{ MW}$$

$$P'_2 = 121 \text{ MW}$$

So, P_1 decreases

F when $P_1 = 175 \text{ MW}$ and $P_2 = 115 \text{ MW}$

$$F_1 = 0.1 \times 1.028 \times 175^2 + 40 \times 175 + 120 = 10268.2 \text{ Rs/hr}$$

$$F_2 = 0.2 \times 115^2 + 30 \times 115 + 100 = 6195 \text{ Rs/hr}$$

$$F = F_1 + F_2 = 16463 \text{ Rs/hr}$$

$$F'_1 = 0.1 \times 1.1308 \times 169^2 + 40 \times 169 + 120 = 10109.6$$

$$F'_2 = 0.2 \times 121^2 + 30 \times 121 + 100 = 6658.2$$

$$F' = F'_1 + F'_2 = 16767.8 \text{ Rs/hr}$$

So, F increase

Hence, the correct option is (A).

Question 22 Electrical machine (DC machine)

The four stator conductors (A, A', B and B') of a rotating machine are carrying DC currents of the same value, the directions of which are shown in the figure (i). The rotor coils $a-a'$ and $b-b'$ are formed by connecting the back ends of conductors ' a ' and ' a' ' and ' b ' and ' b' ', respectively, as shown in figure (ii).

The emf induced in coil $a-a'$ and coil $b-b'$ are denoted by $E_{a-a'}$ and $E_{b-b'}$, respectively. If the rotor is rotated at uniform angular speed ω rad/s in the clockwise direction then which of the following correctly describes the $E_{a-a'}$ and $E_{b-b'}$?

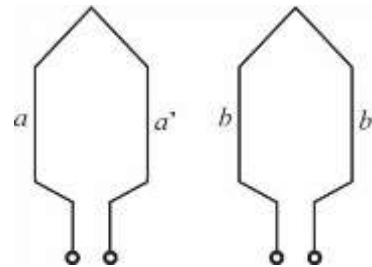
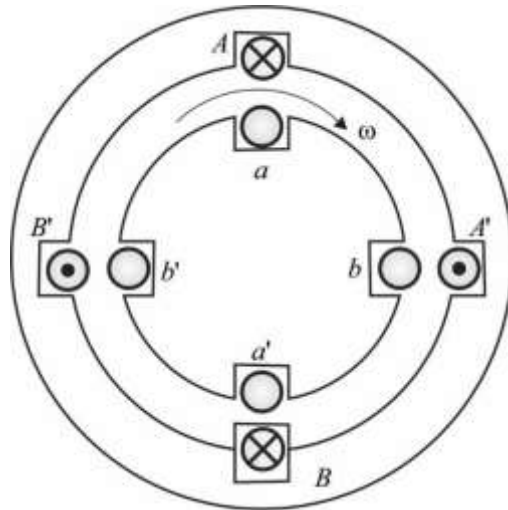
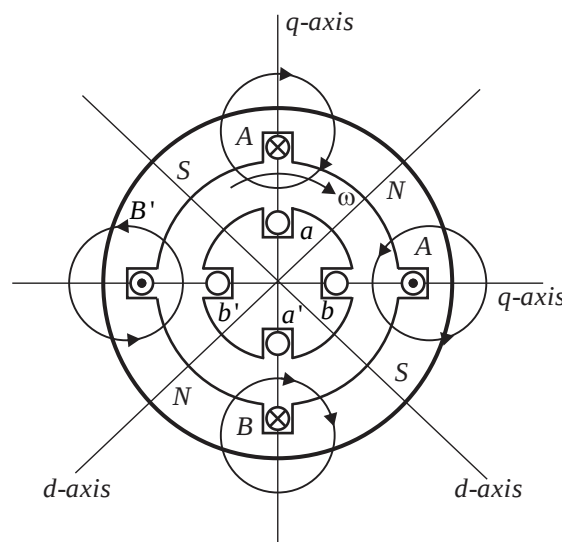


Fig. (i) Cross-sectional view Fig. (ii) Rotor winding connection diagram

- (A) $E_{a-a'}$ and $E_{b-b'}$ have finite magnitudes and are in the same phase.
- (B) $E_{a-a'}$ and $E_{b-b'}$ have finite magnitudes with $E_{b-b'}$ leading $E_{a-a'}$.
- (C) $E_{a-a'}$ and $E_{b-b'}$ have finite magnitudes with $E_{a-a'}$ leading $E_{b-b'}$.
- (D) $E_{a-a'} = E_{b-b'} = 0$.

Ans. (D)
Sol.



At this instant, coil aa' and bb' are along q -axis, so there is no induced emf in both coils

$$E_{aa'} = E_{bb'} = 0.$$

Hence, the correct option is (D).

Question 23 Power Electronics (chopper and commutation)

The chopper circuit shown in figure (i) feeds power to a 5 A DC constant current source. The switching frequency of the chopper is 100 kHz. All the components can be assumed to be ideal. The gate signals of switches S_1 and S_2 are shown in the figure (ii). Average voltage across the 5 A current source is

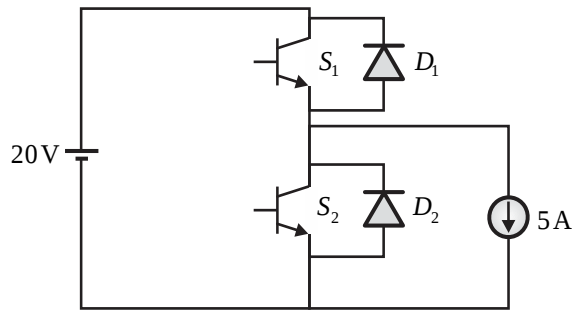


Fig. (i)

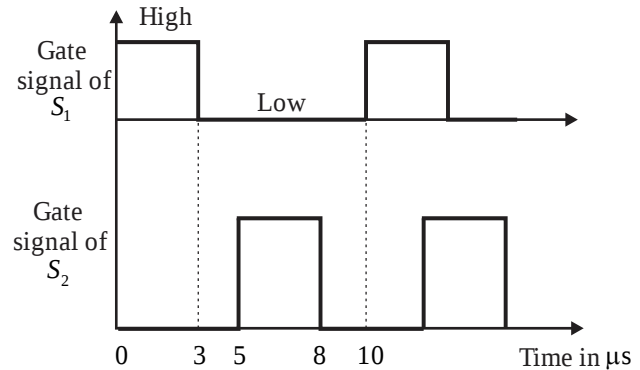


Fig. (ii)

- (A) 10 V
(C) 12 V

- (B) 6 V
(D) 20 V

Ans. (B)

Sol.

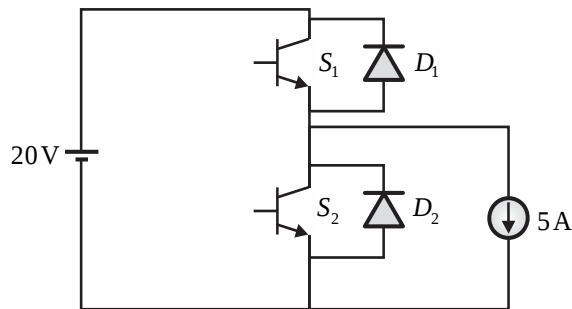


Fig. (i)

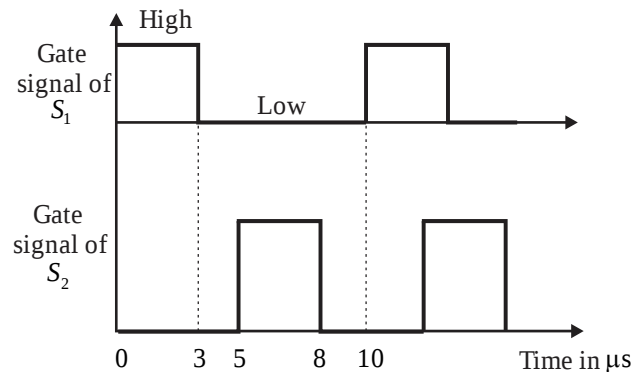
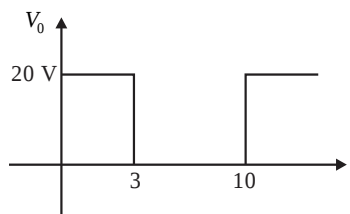


Fig. (ii)

When switch S_1 ON $\rightarrow v_o = v_s = 200 \text{ V}$

D_2 ON $\rightarrow v_o = 0 \text{ Volt}$

S_2 ON $\rightarrow v_o = 0 \text{ Volt}$ (no energy stored)

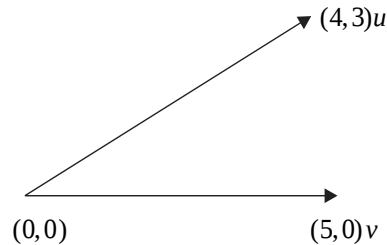


$$V_{0(avg)} = \frac{20 \times 3}{10} = 6 \text{ Volt}$$

Hence, the correct option is (B).

Question 24 Mathematics (Linear algebra)

In the figure the vectors u and v are related as : $Au = v$ by a transformation matrix A . The correct choice of A is



(A) $\begin{bmatrix} \frac{4}{5} & \frac{3}{5} \\ -\frac{3}{5} & \frac{4}{5} \end{bmatrix}$

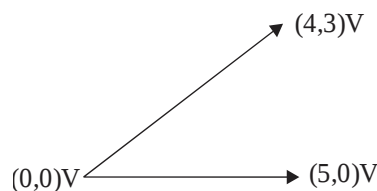
(B) $\begin{bmatrix} \frac{4}{5} & -\frac{3}{5} \\ \frac{3}{5} & \frac{4}{5} \end{bmatrix}$

(C) $\begin{bmatrix} \frac{4}{5} & \frac{3}{5} \\ \frac{3}{5} & \frac{4}{5} \end{bmatrix}$

(D) $\begin{bmatrix} \frac{4}{5} & -\frac{3}{5} \\ \frac{3}{5} & -\frac{4}{5} \end{bmatrix}$

Ans. (A)

Sol. Given : $Au = v$



Considering option (A),

$$\begin{bmatrix} \frac{4}{5} & \frac{3}{5} \\ -\frac{3}{5} & \frac{4}{5} \end{bmatrix} \begin{bmatrix} 4 \\ 3 \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \end{bmatrix}$$

Hence, option (A) satisfies the relation $Au = v$.

Considering option (B),

$$\begin{bmatrix} \frac{4}{5} & -\frac{3}{5} \\ \frac{3}{5} & \frac{4}{5} \end{bmatrix} \begin{bmatrix} 4 \\ 3 \end{bmatrix} = \begin{bmatrix} \frac{7}{5} \\ \frac{24}{5} \end{bmatrix}$$

Hence, option (B) does not satisfy the relation $Au = v$.

Considering option (C),

$$\begin{bmatrix} \frac{4}{5} & \frac{3}{5} \\ \frac{3}{5} & \frac{4}{5} \end{bmatrix} \begin{bmatrix} 4 \\ 3 \end{bmatrix} = \begin{bmatrix} 5 \\ \frac{24}{5} \end{bmatrix}$$

Hence, option (C) does not satisfy the relation $Au = v$.

Considering option (D),

$$\begin{bmatrix} \frac{4}{5} & -\frac{3}{5} \\ \frac{3}{5} & -\frac{4}{5} \end{bmatrix} \begin{bmatrix} 4 \\ 3 \end{bmatrix} = \begin{bmatrix} \frac{7}{5} \\ 0 \end{bmatrix}$$

Hence, option (D) does not satisfy the relation $Au = v$.

Hence, the correct option is (A).

Question 25 Mathematics (numerical methods)

One million random numbers are generated from a statistically stationary process with a Gaussian distribution with mean zero and standard deviation σ_0 .

The σ_0 is estimated by randomly drawing out 10,000 numbers of samples (x_n). The estimates $\hat{\sigma}_1, \hat{\sigma}_2$ are computed in the following two ways.

$$\hat{\sigma}_1^2 = \frac{1}{10000} \sum_{n=1}^{10000} x_n^2, \quad \hat{\sigma}_2^2 = \frac{1}{9999} \sum_{n=1}^{10000} x_n^2$$

Which of the following statements is true?

(A) $E(\hat{\sigma}_2^2) = \sigma_0^2$

(B) $E(\hat{\sigma}_2) = \sigma_0$

(C) $E(\hat{\sigma}_1^2) = \sigma_0^2$

(D) $E(\hat{\sigma}_1) = E(\hat{\sigma}_2)$

Ans. (C)

Sol. Given : $\hat{\sigma}_1^2 = \frac{1}{10000} \sum_{n=1}^{10000} x_n^2, \quad \hat{\sigma}_2^2 = \frac{1}{9999} \sum_{n=1}^{10000} x_n^2$

Here mean, $\sigma_0 E(x^2) = \text{var}(X)$

For the samples in population, then for variance we use the formula :

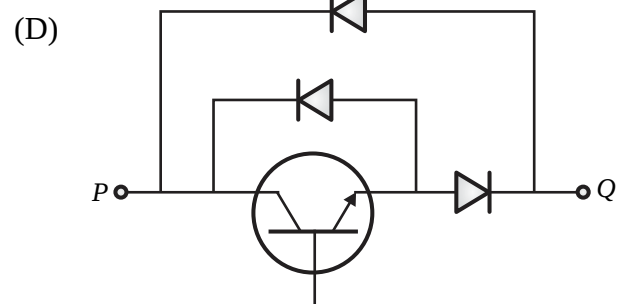
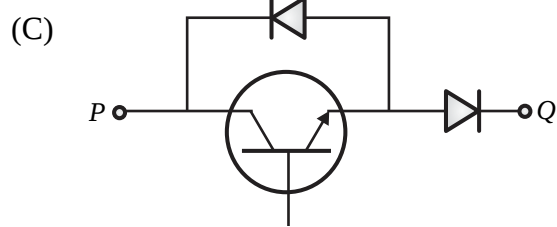
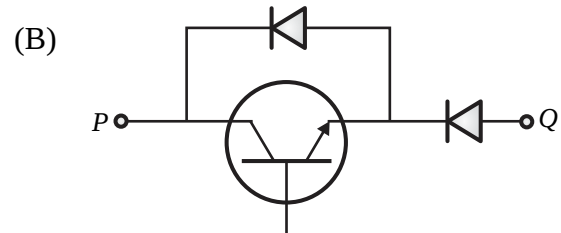
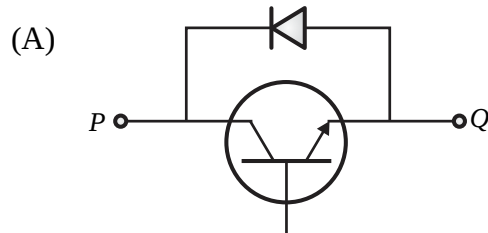
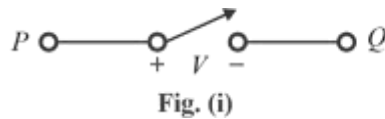
$$E(\sigma_0^2) = \frac{1}{N} \sum (x_n^2) \quad \{\because \text{var} = msv\}$$

Here, $N = 10000$ so $E(\hat{\sigma}_1^2) = \sigma_0^2$

Hence, the correct option is (C).

Question 26 Power electronics (power semiconductor device)

A semiconductor switch needs to block voltage V of only one polarity ($V > 0$) during OFF state as shown in figure (i) and carry current in both directions during ON state as shown in figure (ii). Which of the following switch combination(s) will realize the same?



Ans. (A), (D)

Sol. From given configuration, the current flows in both direction (Bidirectional). The switch also has on drop voltage. The switch configuration in option (A) and (D) can provide bidirectional current. Hence, the correct options are (A) and (D).

Question 27 Signals & Systems (classification of system)

Which of the following statement(s) is/are True?

- (A) If an LTI system is causal, it is stable.
- (B) A discrete time LTI system is causal if and only if its response to a step input $u[n]$ is 0 for $n < 0$.
- (C) If a discrete time LTI system has an impulse response $h(n)$ of finite duration, the system is stable.
- (D) If the impulse response $0 < |h[n]| < 1$ for all n , then the LTI system is stable.

Ans. (B)

Sol. 1. No information about amplitude of $h(n)$ is given. So, option (C) is wrong.
2. $h(n)$ can have any amplitude less than infinity for the LTI system to be stable. So, option (D) is wrong.
3. A causal LTI system can also be unstable. So, option (A) is wrong.
4. If the response to a step input $u[n]$ is 0 for $n < 0$, then the discrete time LTI system will be causal.
So, option (B) is true.

Hence, the correct option is (B).

Question 28 Power system (per unit system)

The bus admittance (Y_{bus}) matrix of a 3-bus power system is given below.

$$\begin{matrix} & \begin{matrix} 1 & 2 & 3 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \end{matrix} & \begin{bmatrix} -j15 & j10 & j5 \\ j10 & -j13.5 & j4 \\ j5 & j4 & -j8 \end{bmatrix} \end{matrix}$$

Considering that there is no shunt inductor connected to any of the buses, which of the following cannot be true?

- (A) Line charging capacitor of finite value is present in all three lines.
- (B) Line charging capacitor of finite value is present in line 2-3 only.
- (C) Line charging capacitor of finite value is present in line 2-3 only and shunt capacitor of finite value is present in bus 1 only.
- (D) Line charging capacitor of finite value is present in line 2-3 only and shunt capacitor of finite value is present in bus 3 only.

Ans. (A), (C)

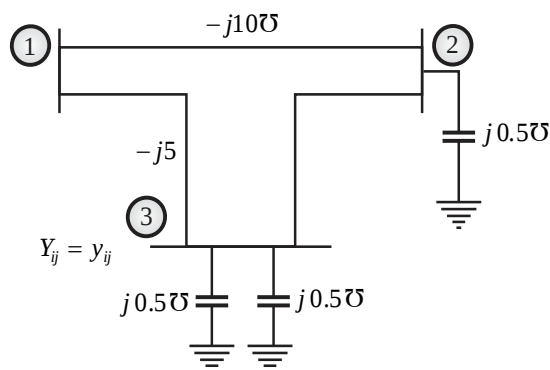
Sol. Given :

$$\begin{matrix} 1 & \begin{bmatrix} -j15 & j10 & j5 \\ -j10 & -j13.5 & j4 \\ j5 & j4 & -j8 \end{bmatrix} & = 0 \\ 2 & & = j0.5 \\ 3 & & = j1 \end{matrix}$$

$\sum R_1 = 0$, so no shunt element at bus-1 is present.

Where as $\sum R_2 = j0.5$ and $\sum R_3 = j1$

So, shunt element are present at bus-2 and 3.



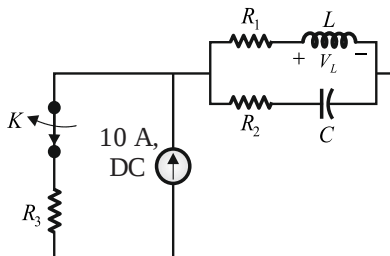
Hence, the correct options are (A) & (C).

Question 29 Network Theory (steady state Analysis)

The value of parameters of the circuit shown in the figure are :

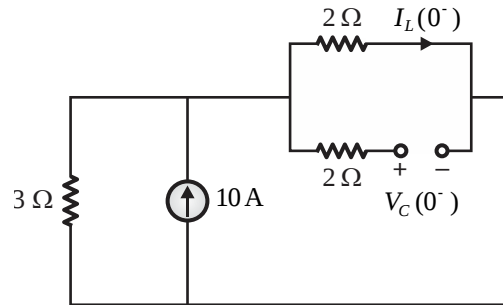
$$R_1 = 2 \, \Omega, R_2 = 2 \, \Omega, R_3 = 3 \, \Omega, L = 10 \, \text{mH}, C = 100 \, \mu\text{F}.$$

For time $t < 0$, the circuit is at steady state with the switch 'K' in closed condition. If the switch is opened at $t = 0$, the value of the voltage across the inductor (V_L) at $t = 0^+$ in Volts is _____ (Round off to 1 decimal place).



Ans. 8 (7.9 to 8.1)

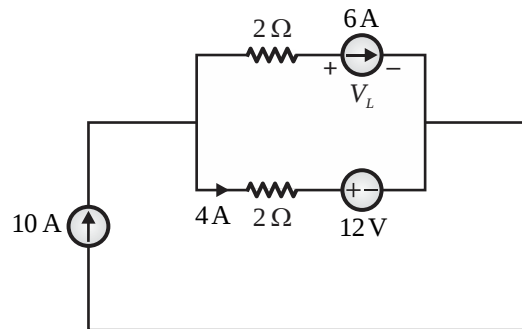
Sol. At $t = 0^-$, switch is closed and circuit is in steady state condition (inductor acts as short circuit and capacitor acts as open circuit).



$$\text{By CDR, } I_L(0^-) = 10 \times \frac{3}{3+2} = 6 \text{ A}$$

$$V_C(0^-) = 6 \times 2 = 12 \text{ V}$$

For $t = 0^+$, switch is opened



Apply KVL,

$$6 \times 2 + V_L - 12 - 4 \times 2 = 0$$

$$V_L = 20 - 12$$

$$V_L = 8 \text{ V}$$

Hence, the correct answer is 8 V.

Question 30 Electrical Machine (DC Machine)

A separately excited DC motor rated 400V, 15A, 1500 rpm drives a constant torque load at rated speed operating from 400 V DC supply drawing rated current. The armature resistance is 1.2Ω . If the supply voltage drops by 10% with field current unaltered, then resultant speed of motor in RPM is _____. (round off to the nearest integer).

Ans. 1343 (1340 to 1345)

Sol. Given : Separately excited DC motor, $\phi = \text{constant}$

$$V_t = 400 \text{ V}, I_{a1} = 15 \text{ A}, R_a = 1.2\Omega, N_1 = 1500 \text{ rpm}, T = \text{constant}$$

If voltage is reduced by 10%

$$V_2 = 0.9V_1$$

$$V_2 = 0.9 \times 400 = 360 \text{ V}$$

$$T = k\phi I_a$$

Since, T and $k\phi$ are constant, $I_a = \text{Constant}$

$$I_{a1} = I_{a2} = 15A$$

$$E_b = V_t - I_a R_a$$

$$k_1 \phi N = V_t - I_a R_a$$

$$\frac{k_1 \phi N_1}{k_1 \phi N_2} = \frac{V_{t_1} - I_{a_1} R_a}{V_{t_2} - I_{a_2} R_a}$$

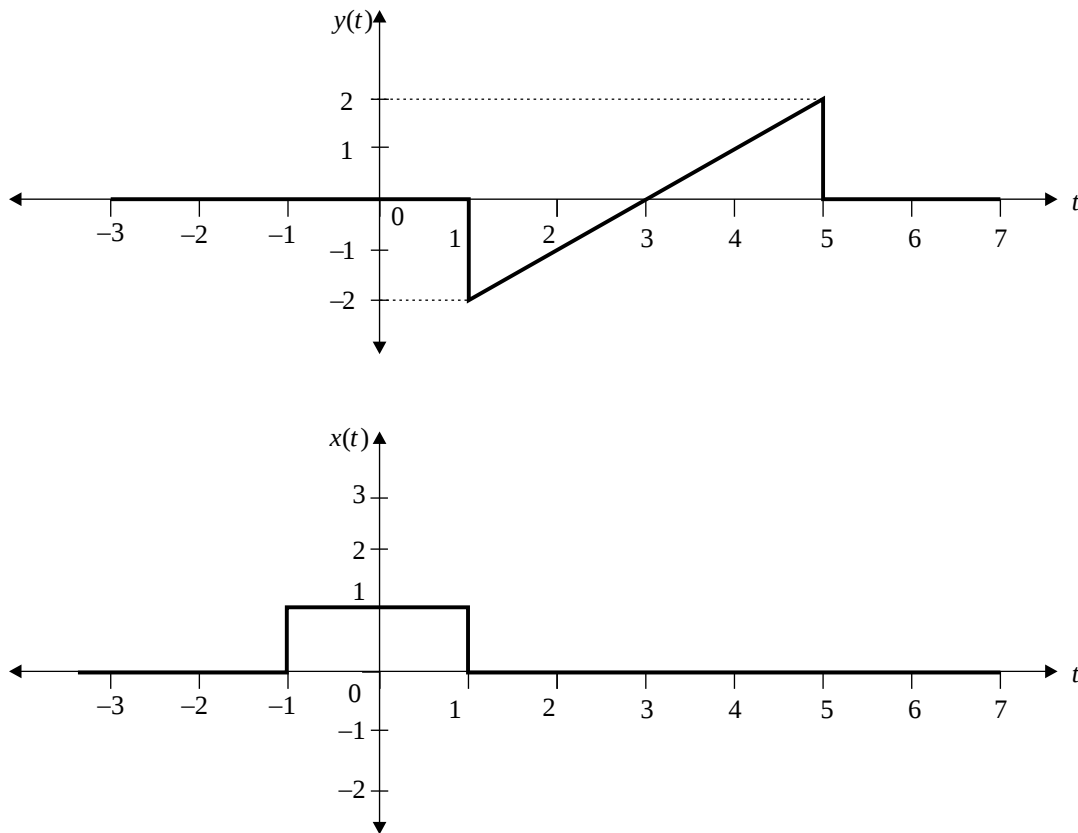
$$\frac{1500}{N_2} = \frac{400 - 15 \times 1.2}{360 - 15 \times 1.2}$$

$$N_2 = 1343 \text{ rpm}$$

Hence, the resultant speed of motor is 1343 rpm.

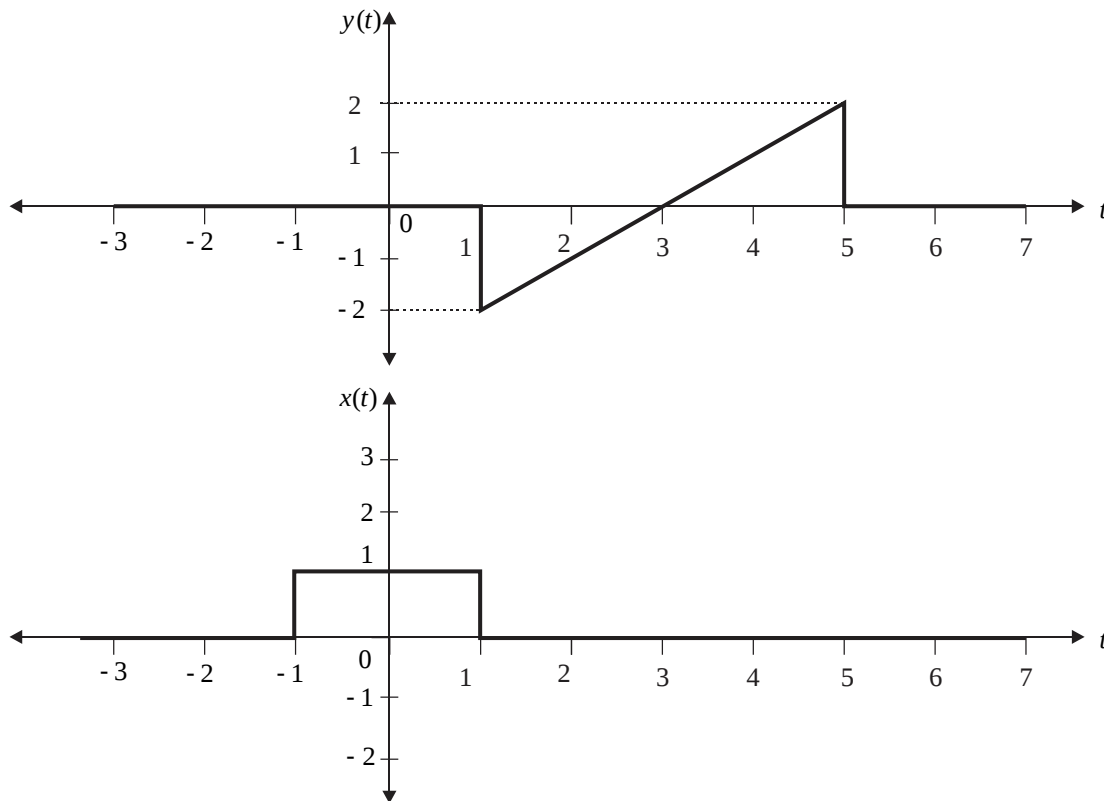
Question 31 Signals & Systems (basic of signal)

For the signals $x(t)$ and $y(t)$ shown in the figure, $z(t) = x(t) * y(t)$ is maximum at $t = T_1$. Then T_1 in seconds is _____. (round off to the nearest integer).



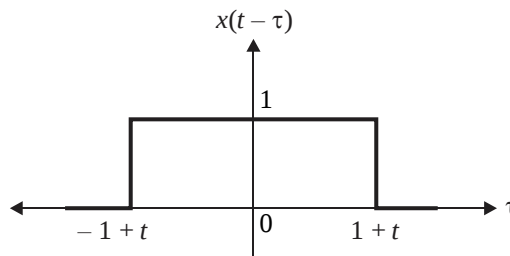
Ans. 4 (4 to 4)

Sol. Given signal $y(t)$ and $x(t)$ is shown in figure below,



Given : $z(t) = x(t) * y(t) = \int_{-\infty}^{\infty} y(\tau) x(t - \tau) d\tau$

$x(t - \tau)$ can be plotted as shown below,



Maximum value of convolution will be obtained when positive portion of $y(\tau)$ is multiplied with $x(t - \tau)$

$$\therefore -1 + t = 5$$

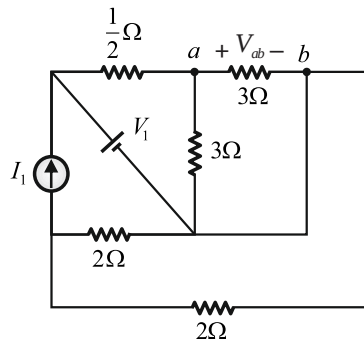
$$t = 4 \text{ sec}$$

At $t = 4$ sec, maximum value of convolution will be obtained.

Hence, the correct answer is 4.

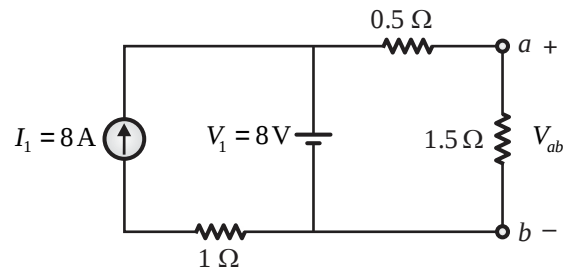
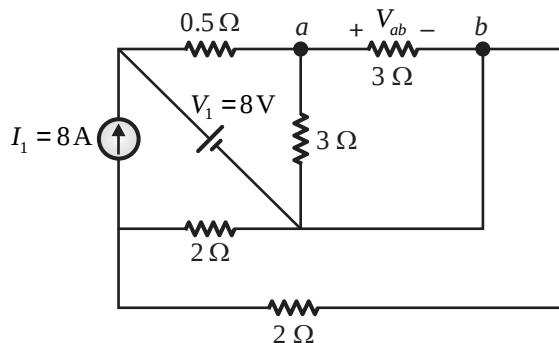
Question 32 Network theory (network theorem)

For the circuit shown in the figure, $V_1 = 8\text{ V}$, DC and $I_1 = 8\text{ A}$, DC. The voltage V_{ab} in Volts is ____
(Round off to 1 decimal place).



Ans. 6 (5.9 to 6.1)

Sol. Given circuit is,



By VDR,

$$V_{ab} = \frac{8 \times 1.5}{1.5 + 0.5} = \frac{8 \times 1.5}{2}$$

$$V_{ab} = 6\text{ V}$$

Hence, the correct answer is 6.

Question 33 Power system (performance of transmission line)

A 50 Hz, 275 kV line of length 400 km has the following parameters :

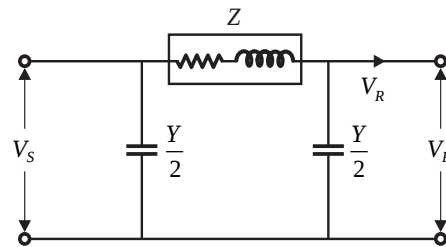
Resistance $R = 0.035\ \Omega/\text{km}$; Inductance $L = 1\text{ mH/km}$; Capacitance $C = 0.01\ \mu\text{F/km}$.

The line is represented by the nominal- π model. With the magnitudes of sending end and the receiving end voltages of the line (denoted by V_S and V_R , respectively) maintained at 275 kV, the phase angle difference (θ) between V_S and V_R required for maximum possible active power to be delivered to the receiving end, in degree is ____ (round off to 2 decimal places).

Ans. 83.64 (83 to 84)

Sol. Given : $R = 0.035 \times 400 = 14\ \Omega$

$$L = 1 \times 10^{-3} \times 400 = 0.4\text{ H}$$



$$Z = 14 + 2\pi \times 50 \times j0.4 = (14 + j125.67) \Omega$$

$$S_R = V_R I_R^*$$

$$P_R = \frac{|V_s||V_R|}{|B|} \cos(\beta - \theta) - \frac{|A||V_R|^2}{|B|} \cos(\beta - \alpha)$$

For maximum power P_R

$$\cos(\beta - \theta) = 1$$

$$\beta = \theta$$

$$B = |Z| = 126.45 \angle 83.64^\circ$$

$$\therefore \theta = 83.64^\circ$$

Question 34 Mathematics (differential equations)

In the following differential equation, the numerically obtained value of $y(t)$, at $t = 1$, is _____ (Round off to 2 decimal places).

$$\frac{dy}{dt} = \frac{e^{-\alpha t}}{2 + \alpha t}, \alpha = 0.01 \text{ and } y(0) = 0$$

Ans. 0.5 (0.48 to 0.52)

Sol. Given : $\frac{dy}{dt} = \frac{e^{-\alpha t}}{2 + \alpha t} \dots(i)$

$$\alpha = 0.01$$

$$y(0) = 0 \text{ (initial)}$$

$$f(t, y) = \frac{dy}{dt} = \frac{e^{-0.01t}}{2 + 0.01t}$$

Euler's method : Step size $h = t$ (given)

$$y_1 = y_0 + hf(t, y) \quad (\text{at } t_0 = 0, y_0 = 0)$$

$$= 0 + 1 \left[\frac{e^{-0.01 \times 0}}{2 + 0.01 \times 0} \right]$$

$$= 0 + 1 \times \frac{1}{2} = \frac{1}{2} = 0.5$$

$$\text{So, at } t = 1, y_1 = 0.5$$

Hence, the correct answer is 5.

Question 35 Mathematics (numerical methods)

Three points in the x - y planes are $(-1, 0.8)$, $(0, 2.2)$ and $(1, 2.8)$. The value of the slope of the best fit straight line in the least square sense is _____. (round off to two decimal places).

Ans. 1 (0.9 to 1.1)**Sol.** Given points in x - y planes are,

x	-1	0	1
y	0.8	2.2	2.8

Let the straight line, $y = ax + b$

By least square approximation, then by regression line method.

$$\sum y_i = a \sum x_i + b n \quad \dots(i)$$

$$\sum x_i y_i = a \sum x_i^2 + b \sum x_i \quad \dots(ii)$$

x	y	x^2	xy
-1	0.8	1	-0.8
0	2.2	0	0
1	2.8	1	2.8
$\sum x = 0$	$\sum y = 5.8$	$\sum x^2 = 2$	$\sum xy = 2$

Substituting the values in equations (i) and (ii),

$$5.8 = a(0) + 3b$$

$$2 = a(2) + 0(b)$$

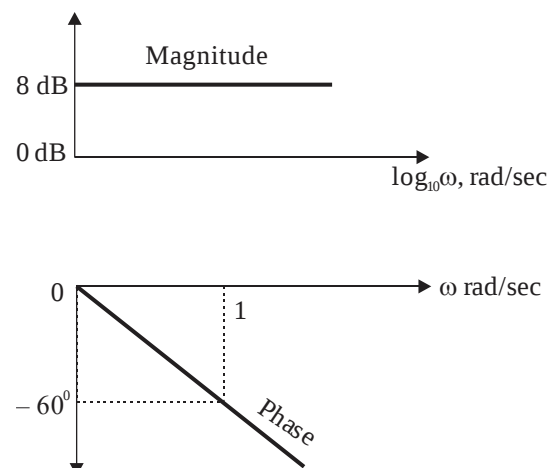
$$a = 1, b = 1.9$$

The value of slope $a = 1$.

Hence, the correct answer is 1.

Q.36 to Q.65 Carry Two Marks Each**Question 36 Control system (bode plot)**

Magnitude and phase plots of an LTI system are shown in the figure. The transfer function of the system is



(A) $2.51e^{-0.032s}$

(B) $\frac{e^{-2.514s}}{s+1}$

(C) $1.04e^{-2.514s}$

(D) $2.51e^{-1.047s}$

Ans. (D)**Sol.** From the given bode plot

Let $T(s) = Ke^{-sT}$

Where, $T \rightarrow$ Transportation lag.From bode plot, $20\log K = 8\text{dB}$

$K = 2.511$

At $\omega = 1 \text{ rad/sec}$, $\phi = -60^\circ$

From transfer function, $\phi = -\omega T \times \frac{180^\circ}{\pi}$

$$-60^\circ = -T \times \frac{180^\circ}{\pi}$$

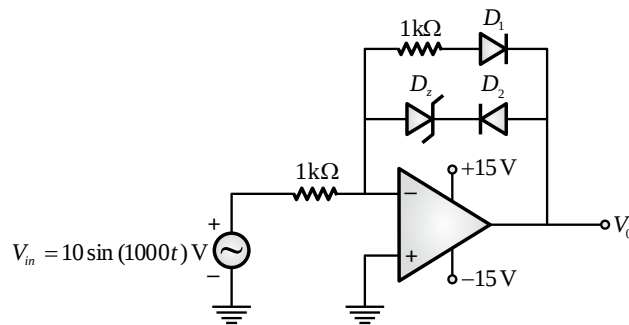
$T = 1.047$

$T(s) = 2.511e^{-1.047s}$

Hence, the correct option is (D).

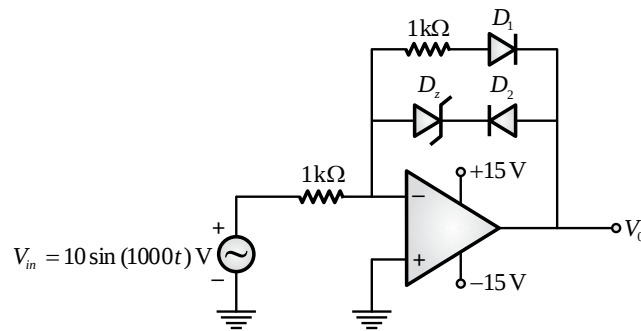
Question 37 Analog Electronics (Operations amplifier)

Consider the Op-Amp based circuit shown in the figure. Ignore the conduction drops of diodes D_1 and D_2 . All the components are ideal and the breakdown voltage of the Zener is 5 V. Which of the following statements is true?

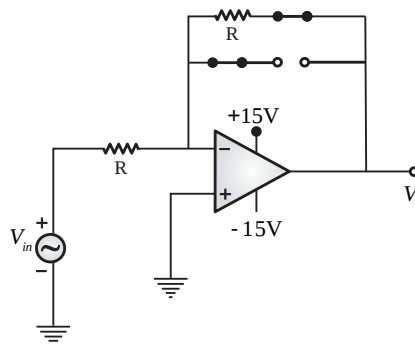


- (A) The maximum and minimum values of the output voltage V_o are +15 V and -10 V, respectively.
- (B) The maximum and minimum values of the output voltage V_o are +5 V and -15 V, respectively.
- (C) The maximum and minimum values of the output voltage V_o are +10 V and -5 V, respectively.
- (D) The maximum and minimum values of the output voltage V_o are +5 V and -10 V, respectively.

Ans. (D)**Sol.** Given circuit is,



For positive half cycle the diodes D_1 and D_2 are forward bias and D_2 is reverse biased and the circuit is shown below,

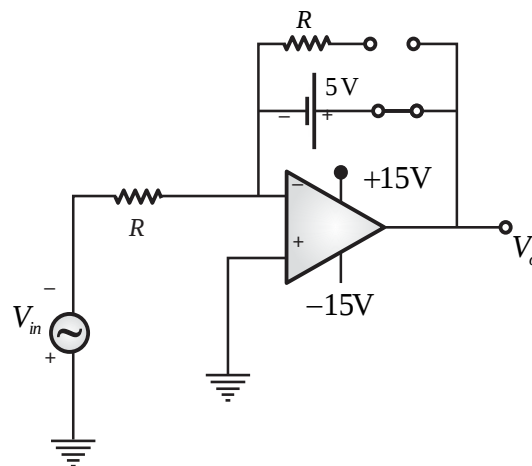


When $V_{in} = 10V$,

$$V_o = \frac{-R}{R} \times 10 = -10V$$

$\therefore V_o$ will be $-10V$.

For negative half cycle, diode D_2 is forward bias and D_1 , D_2 is reverse bias. Zener diode is in breakdown region and the circuit is shown below,



$$\therefore V_o = 5V$$

$$\Rightarrow V_{o_{max}} = 5V \text{ and } V_{o_{min}} = -10V$$

Hence, the correct option is (D).

Question 38 Control system (bode plot)

Consider a lead compensator of the form $K(s) = \frac{1 + \frac{s}{\alpha}}{1 + \frac{s}{\alpha\beta}}$, $\beta > 1$, $\alpha > 0$.

The frequency at which this compensator produces maximum phase lead is 4 rad/sec. At this frequency, the gain amplification provided by the controller, assuming asymptotic Bode magnitude plot of $K(s)$, is 6 dB. The values of α, β respectively

(A) 1, 16

(B) 2, 4

(C) 3, 5

(D) 2.66, 2.25

Ans. (B)

Sol. Given : $K(s) = \frac{1 + \frac{s}{\alpha}}{1 + \frac{s}{\alpha\beta}}$

$$K(s) = \frac{s + \alpha}{\alpha} \times \frac{\alpha\beta}{s + \alpha\beta}$$

$$K(s) = \frac{\beta(s + \alpha)}{(s + \alpha\beta)}$$

$$\sqrt{\alpha \times \alpha\beta} = 4$$

$$\alpha\sqrt{\beta} = 4 \quad \dots(i)$$

$$\text{Also, } \alpha\beta > \alpha \quad \dots(ii)$$

$$K(j\omega) = \frac{\beta(j\omega + \alpha)}{j\omega + \alpha\beta}$$

$$|K(j\omega)| = \frac{\beta\sqrt{\omega^2 + \alpha^2}}{\sqrt{\omega^2 + (\alpha\beta)^2}}$$

$$\text{Given : } 20 \log_{10} |K(j\omega)| = 6 \text{ dB at } 4 \text{ rad/sec.}$$

$$\Rightarrow |K(j\omega)| = 2$$

$$\Rightarrow \frac{\beta\sqrt{\omega^2 + \alpha^2}}{\sqrt{\omega^2 + (\alpha\beta)^2}} = 2$$

$$\Rightarrow \frac{\beta\sqrt{16 + \alpha^2}}{\sqrt{16 + (\alpha\beta)^2}} = 2$$

$$\Rightarrow \frac{\beta^2(16 + \alpha^2)}{16 + (\alpha\beta)^2} = 4$$

$$\Rightarrow \beta^2(16 + \alpha^2) = 64 + 4(\alpha\beta)^2$$

$$\Rightarrow \alpha^2 \beta^2 + 16\beta^2 = 64 + 4(\alpha\beta)^2$$

$$\Rightarrow 16\beta^2 = 64 + 3(\alpha\beta)^2 \quad \dots\dots(iii)$$

Only the values $\alpha = 2$ and $\beta = 4$ satisfies the conditions (i),(ii) and (iii)

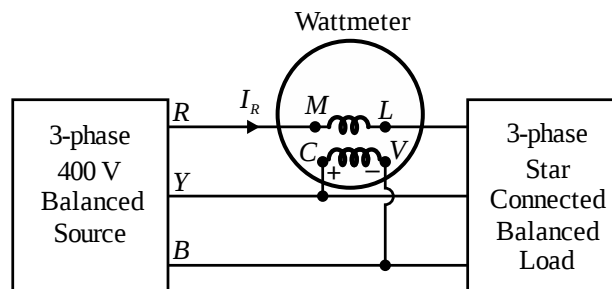
Hence, the correct option is (B).

Key Point :

1. If $G(s) = \frac{k(s+z)}{s+p}$ is the transfer function of a lead compensator then the frequency at which this compensator provides maximum phase lead is, $\omega_m = \sqrt{p \times z}$ rad/sec.
2. If $G(s) = \frac{k(s+z)}{s+p}$ has to act as a lead compensator then p must be greater than z i.e. $p > z$.

Question 39 Measurements (measurements of energy and power)

A 3 - phase, star connected, balanced load is supplied from a 3 phase, 400 V (rms), balanced voltage source with phase sequence R-Y-B, as shown in the figure. If the wattmeter reading is – 400 W and the line current is $I_R = 2A$ (rms), then the power factor of the load per phase is



(A) Unity

(C) 0.866 leading

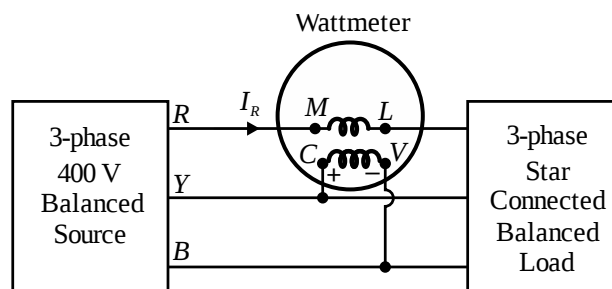
(B) 0.5 leading

(D) 0.707 lagging

Ans. (C)

Sol. Given : $V_L = 400V$ (rms), Phase sequence $\rightarrow RYB$, Wattmeter reading = – 400 W

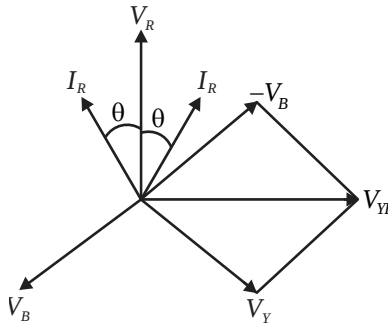
Line current, $I_R = 2A$ (rms)



$$\text{Wattmeter Reading} = V_{pc} I_{cc} \cos V_{pc} I_{cc}$$

$$V_{pc} = V_{YB}$$

$$I_{CC} = I_R = 2A$$



$$\bar{V}_{YB} = \bar{V}_Y - \bar{V}_B$$

If balanced lagging load, $P_C = V_{YB} I_R \cos(90 - \theta)$

If balanced leading load, $P_C = V_{YB} I_R \cos(90 + \theta)$

But it is given that power is negative, so Load power factor is leading as $\cos(90 + \theta) = -ve$

$$-400 = 400 \times 2 \cos(90 + \theta)$$

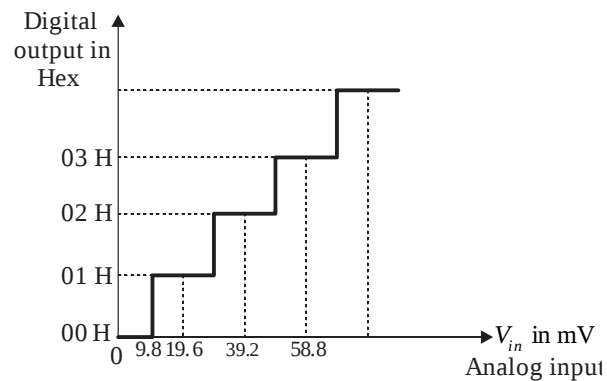
$$\theta = 30^\circ$$

Power factor = $\cos 30^\circ = 0.866$ leading

Hence, the correct option is (C).

Question 40 Analog Electronics (ADC and DAC)

An 8-bit ADC converts analog voltage in the range of 0 to +5 V to the corresponding digital code as per the conversion characteristics shown in figure. For $V_{in} = 1.9922$ volt, which of the following digital output, given in hex, is true?



(A) 64 H

(B) 65 H

(C) 66 H

(D) 67 H

Ans. (C)

Sol. Given : $n = 8$ (8-bit ADC), $V_{max} = 5$ V .

$$\text{Step size} = \frac{V_{max}}{2^n - 1} = \frac{5}{2^8 - 1} = \frac{5}{255}$$

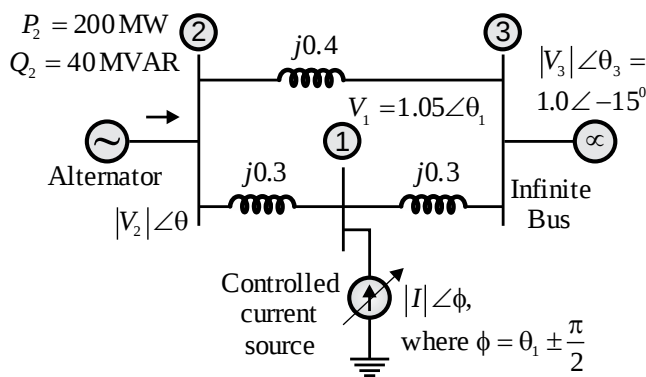
Analog input, $V_{in} = 1.9922$ V

Digital output will be, $\frac{1.9922 \times 255}{5} = 101.592 \approx (102)_{10} = (66)_H$

Hence, the correct option is (C).

Question 41 Power system (per Unit system)

The three-bus power system shown in the figure has one alternator connected to bus 2 which supplies 200 MW and 40 MVAR power. Bus 3 is infinite bus having a voltage of magnitude $|V_3| = 1.0$ p.u. and angle of -15° . A variable current source, $|I| \angle \phi$ is connected at bus 1 and controlled such that the magnitude of the bus 1 voltage is maintained at 1.05 p.u. and the phase angle of the source current, $\phi = \theta_1 \pm \frac{\pi}{2}$, where θ_1 is the phase angle of the bus 1 voltage. The three buses can be categorized for load flow analysis as



- | | |
|-------------------------|---------------------------|
| (A) Bus 1 Slack bus | (B) Bus 1 P - $ V $ bus |
| Bus 2 P - $ V $ bus | Bus 2 P - $ V $ bus |
| Bus 3 P - Q bus | Bus 3 Slack bus |
| (C) Bus 1 P - Q bus | (D) Bus 1 P - $ V $ bus |
| Bus 2 P - Q bus | Bus 2 P - Q bus |
| Bus 3 Slack bus | Bus 3 Slack bus |

Ans. (D)

Sol. Bus (1) $\rightarrow P, V \rightarrow PV$ bus

Bus (2) $\rightarrow P, Q \rightarrow PQ$ bus

Bus (3) $\rightarrow V, S \rightarrow Slack$ bus

Hence, the correct option is (D).

Question 42 Mathematics (integration and differentiation)

Consider the following equation in a 2-D real-space.

$$|x_1|^p + |x_2|^p = 1 \text{ for } p > 0$$

Which of the following statement(s) is/are true?

- (A) When $p = 2$, the area enclosed by the curve is π .

(B) When p tends to ∞ , the area enclosed by the curve tends to 4.

(C) When p tends to 0, the area enclosed by the curve is 1.

(D) When $p = 1$, the area enclosed by the curve is 2.

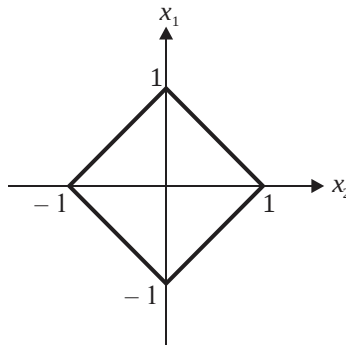
Ans. (A), (B), (D)

Sol. Given equation in a 2-D real space is,

$$|x_1|^p + |x_2|^p = 1, p > 0$$

Let $p = 1$,

$$|x_1| + |x_2| = 1 \text{ is a square}$$



$$\therefore \text{Area} = 4 \times \frac{1}{2} \times 1 \times 1 = 2$$

Let $p = 2$,

$$|x_1|^2 + |x_2|^2 = 1$$

$$x_1^2 + x_2^2 = 1 \text{ which is a circle of radius 1.}$$

$$\therefore \text{Area} = \pi \times 1^2 = \pi$$

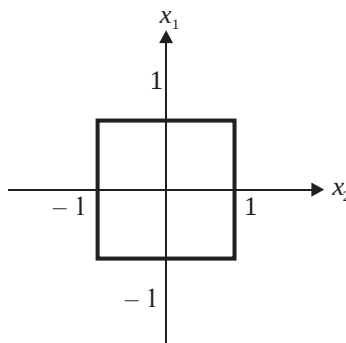
For $p = 0$, the curve does not exist.

Let $p = \infty$,

$$|x_1|^p + |x_2|^p = 1$$

If $x_1 < 1$, then $x_2 = 1$ or -1 .

It will form square of side 2.

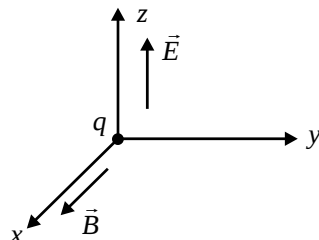


$$\text{Area} = (\text{Side})^2 = 2^2 = 4$$

Hence, the correct options are (A), (B) & (D).

Question 43 MTA (Marks To All) Electromagnetic field (magnetostatic)

In the figure, the electric field E and magnetic field B point to x and z directions, respectively, and have constant magnitude. A positive charge ' q ' is released from rest at the origin. Which of the following statement(s) is/are true.

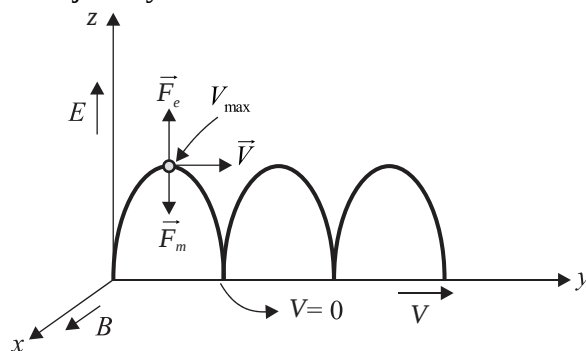


- (A) The charge will move in the direction of z with constant velocity.
- (B) The charge will always move on the y - z plane only.
- (C) The trajectory of the charge will be a circle.
- (D) The charge will progress in the direction of y .

Ans. (B), (D)

Sol. Net force applied on charge, $\vec{F}_{total} = \vec{F}_e + \vec{F}_m = q\vec{E} + q(\vec{V} \times \vec{B})$

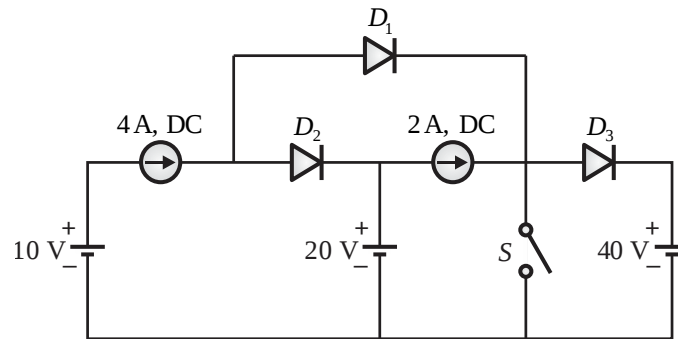
Initially charge is at rest. No magnetic force is experienced, due to electric field charge moves in the z direction with increasing velocity. Now because of increasing v $\vec{F}_m$ increases in perpendicular of velocity vector and creates a cycloid trajectory.



“There is miss match between statement and figure. As per the figure only (B) and (D) are the correct answer. Must go for Marks To All”.

Question 44 Analog Electronics (diode circuit and application)

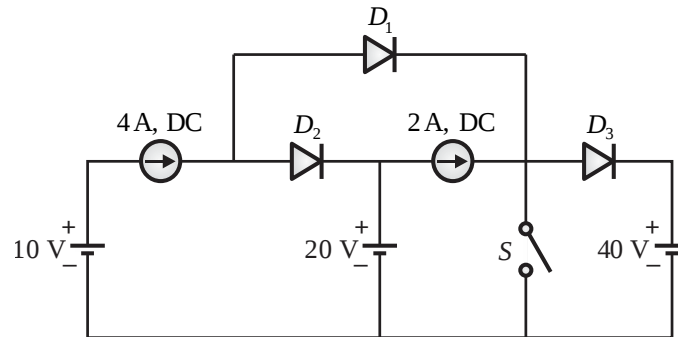
All the elements in the circuit shown in the following figure are ideal. Which of the following statements is/are true?



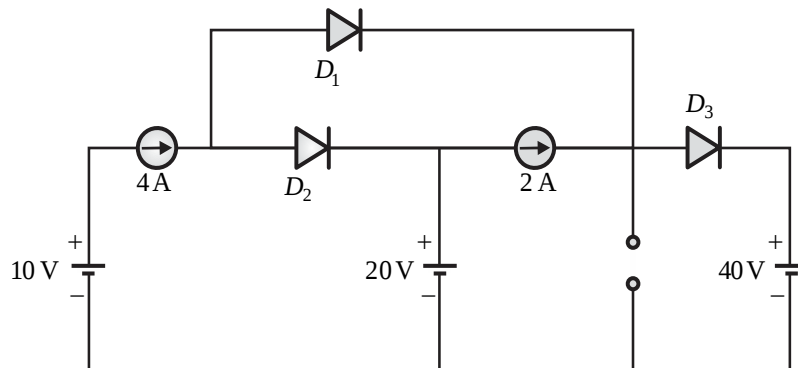
- (A) When switch S is ON, both D_1 and D_2 conduct and D_3 is reverse biased
(B) When switch S is ON, D_1 conducts and both D_2 and D_3 are reverse biased
(C) When switch S is OFF, D_1 is reverse biased and both D_2 and D_3 conduct
(D) When switch S is OFF, D_1 conducts, D_2 is reverse biased and D_3 conducts

Ans. (B), (C)

Sol. Given circuit is shown below,

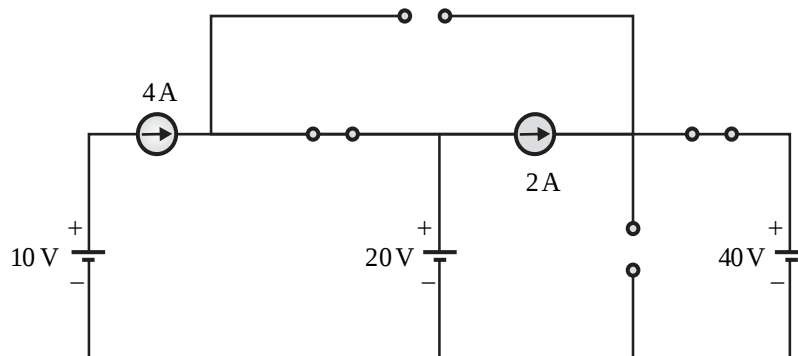


When switch is OFF, the given circuit looks as shown below



Assume D_1 is reverse biased, then the 4 A current flows through D_2 and 2A flows through D_3 .

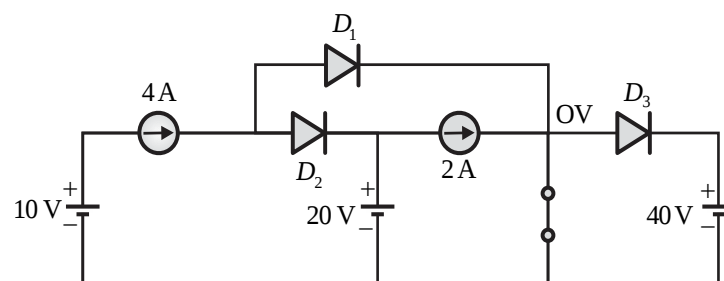
So D_2 and D_3 will be forward biased.



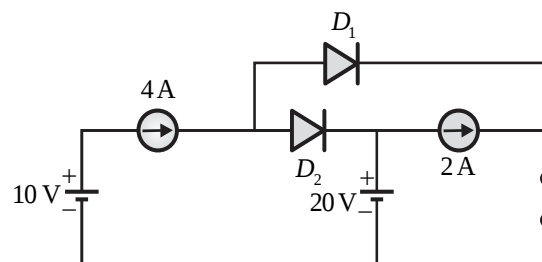
From the above figure, D_1 is reverse biased and our assumption is correct.

Therefore, option (B) is correct.

When switch is ON, the circuit looks as shown below



From the above circuit, D_3 is OFF



Now D_1 and D_2 both cannot be reverse biased together as it violates KCL. If D_2 is forward biased and D_1 is reverse biased, then 4 A current flows from lower potential to higher potential which is not possible.

Hence, D_1 will be forward biased and D_2 will be reverse biased.

Therefore, option (C) is also correct.

Hence, the correct options are (B) and (C).

Question 45 Mathematics (probability and statistics)

The expected number of trials for first occurrence of a “head” in a biased coin is known to be 4. The probability of first occurrence of a “head” in the second trial is _____ (Round off to 3 decimal places).

Ans. 0.187 (0.187 to 0.188)

Sol.

X. No. of trials	1	2	3	
------------------	---	---	---	--

$p(x)$	p	$(1-p)p$	$(1-p)^2 p$	----
--------	-----	----------	-------------	------

By Geometrical distribution,

$$E(x) = \frac{1}{p}$$

$$V(x) = \frac{q}{p^2}$$

$$E(x) = 4 = \frac{1}{p} \quad (\text{Given})$$

$$q = 1 - \frac{1}{4} = \frac{3}{4}$$

$$P(H) = \frac{1}{4},$$

$$P(T) = \frac{3}{4}$$

Required probability, $P(E) = P(TH) = \frac{3}{4} \times \frac{1}{4} = \frac{3}{16} = 0.1875$

Hence, the correct answer is 0.187.

Question 46 Control system (state space analysis)

Consider the state-space description of an LTI system with matrices

$$A = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, C = [3 \quad -2], D = 1$$

For the input, $\sin(\omega t)$, $\omega > 0$, the value of ω for which the steady-state output of the system will be zero, is _____. (round off to the nearest integer)

Ans. 2 (2 to 2)

Sol. Given :

$$A = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, C = [3 \quad -2] \text{ and } D = 1$$

Transfer function is given by,

$$T.F. = C[sI - A]^{-1}B + D$$

$$[sI - A] = \begin{bmatrix} s & -1 \\ 1 & s+2 \end{bmatrix}$$

$$[sI - A]^{-1} = \begin{bmatrix} s & -1 \\ 1 & s+2 \end{bmatrix}^{-1}$$

$$[sI - A]^{-1} = \frac{1}{s(s+2)+1} \begin{bmatrix} s+2 & 1 \\ -1 & s \end{bmatrix}$$

$$T.F. = [3 \quad -2] \frac{1}{s(s+2)+1} \begin{bmatrix} s+2 & 1 \\ -1 & s \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} + 1$$

$$T.F. = \frac{1}{s^2+2s+1} [3 \quad -2] \begin{bmatrix} s+2 & 1 \\ -1 & s \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} + 1$$

$$T.F. = \frac{1}{s^2+2s+1} [3 \quad -2] \begin{bmatrix} 1 \\ s \end{bmatrix} + 1$$

$$T.F. = \frac{3-2s}{s^2+2s+1} + 1$$

$$T.F. = \frac{s^2+4}{s^2+2s+1}$$

$$H(s) = \frac{s^2+4}{s^2+2s+1}$$

$$s = j\omega$$

$$H(j\omega) = \frac{4-\omega^2}{1+2j\omega-\omega^2}$$

Steady state output of system is zero

$$4-\omega^2 = 0$$

$$4 = \omega^2$$

$$\omega = 2 \text{ rad/sec}$$

Hence, the correct answer is 2.

Question 47 Electrical machine (3 phase synchronous machine)

A 3-phase synchronous motor with synchronous impedance of $0.1 + j0.3$ per unit per phase has a static stability limit of 2.5 per unit. The corresponding excitation voltage in per unit is _____ (round off to 2 decimal places).

Ans. 1.58 (1.58 to 1.59)

Sol. Given : $\bar{Z}_s = 0.1 + j0.3 \text{ pu} = 0.316 \angle 71.56^\circ$

$$P_{em} = 2.5 \text{ pu}$$

$$E = ?$$

$$\text{Let } V = 1 \text{ pu}$$

Power output of synchronous motor

$$P_{0_{\max}} = \frac{EV}{Z_s} - \frac{E^2}{Z_s^2} R_G$$

$$2.5 = \frac{E \times 1}{0.316} - \frac{E^2}{0.316^2} \times 0.1, \quad E = 1.58 \text{ pu}$$

Hence, the correct answer is 1.58.

Question 48 Electrical machine (3 phase induction motor)

A 3-phase, 415 V, 50 Hz, 6 pole, 960 RPM, 4 HP squirrel cage induction motor drives a constant torque load at rated speed operating from rated supply and delivering rated output. If the supply voltage and frequency are reduced by 20%, the resultant speed of the motor in RPM (neglecting the stator leakage impedance and rotational losses) is _____. (round off to the nearest integer).

Ans. 760 (760 to 760)**Sol. Given :** 4HP Induction motor

$$V_t = 415 \text{ V}, f_s = 50 \text{ Hz}, P = 6, N_r = 960 \text{ rpm}, T_L = \text{Constant}$$

V and f are reduced by 20 %

Hence, $\frac{V}{f}$ is constant

$$T = \frac{3}{\omega_s} \frac{V^2}{\left(\frac{R_2'}{s}\right) + X_2'^2} \times \frac{R_2'}{s}$$

As we know, motor operates in low slip region

$$\frac{R_2'}{s} \gg X_2' \text{ so, } X_2' \rightarrow \text{Neglect}$$

$$T_d = \frac{3}{\omega_s} \frac{sV^2}{R_2'}$$

For constant load torque $T_d = \text{Constant}$

$$\frac{sV^2}{\omega_s} = \text{Constant}$$

$$\frac{sf^2}{f} = \text{Constant}$$

$$sf = \text{Constant}$$

$$N_s \propto f$$

$$sN_s = \text{Constant}$$

$$N_s - N_r = \text{Constant}$$

$$N_{s_1} - N_{r_1} = N_{s_2} - N_{r_2}$$

$$\frac{120 \times f_1}{P} - 960 = \frac{120 f_2}{P} - N_{r_2}$$

$$\frac{120 \times 50}{6} - 960 = \frac{120 f_2}{6} - N_{r_2}$$

$$1000 - 960 = \frac{120 f_2}{6} - N_{r_2}$$

$$40 = \frac{120 \times f_2}{6} - N_{r_2}$$

$$\therefore f_2 = 0.8 f_1$$

$$f_2 = 0.8 \times 50 = 40 \text{ Hz}$$

$$\therefore 40 = \frac{120 \times 40}{6} - N_{r_2}$$

$$40 = 800 - N_{r_2}$$

$$N_{r_2} = 800 - 40 = 760 \text{ rpm}$$

Hence, the resultant speed of the motor is 760 rpm.

Hence, the correct answer is 760.

Question 49 Signals & Systems (basic of signal)

The period of the discrete time signal $x[n]$ described by the equation below is $N = \underline{\hspace{2cm}}$. (Round off to the nearest integer).

$$x[n] = 1 + 3 \sin\left(\frac{15\pi}{8}n + \frac{3\pi}{4}\right) - 5 \sin\left(\frac{\pi}{3}n - \frac{\pi}{4}\right)$$

Ans. 48 (48 to 48)

Sol. Given : $x(n) = 1 + 3 \sin\left(\frac{15\pi}{8}n + \frac{3\pi}{4}\right) - 5 \sin\left(\frac{\pi}{3}n - \frac{\pi}{4}\right)$

Frequency components of $x(n)$ are : $f_1 = \frac{15\pi}{16\pi} = \frac{15}{16}$ and $f_2 = \frac{\pi}{6\pi} = \frac{1}{6}$

$\therefore$ Time period, $N_1 = 16$ and $N_2 = 6$

Time period of $x(n)$ will be,

$$N = \text{L.C.M}[N_1, N_2] = \text{L.C.M}[16, 6] = 48$$

Hence, the correct answer is 48.

Question 50 Signals & Systems (discrete time convolution)

The discrete time Fourier transform of a signal $x[n]$ is $X(\Omega) = (1 + \cos \Omega)e^{-j\Omega}$. Consider that $x_p[n]$ is a periodic signal of period $N = 5$ such that,

$$x_p[n] = \begin{cases} x[n], & \text{for } n = 0, 1, 2 \\ 0, & \text{for } n = 3, 4 \end{cases}$$

Note that $x_p[n] = \sum_{k=0}^{N-1} a_k e^{j\frac{2\pi}{N}kn}$. The magnitude of the Fourier series coefficient a_3 is $\underline{\hspace{2cm}}$. (Round off to 3 decimal places).

Ans. 0.038 (0.037 to 0.039)

Sol. Method 1

$$X(\Omega) = (1 + \cos \Omega)e^{-j\Omega}$$

$$X(\Omega) = \left(1 + \frac{e^{j\Omega}}{2} + \frac{e^{-j\Omega}}{2}\right) e^{-i\Omega}$$

$$X(\Omega) = e^{-j\Omega} + \frac{1}{2} + \frac{1}{2} e^{-j2\Omega}$$

$$X(\Omega) = \frac{1}{2} + e^{-j\Omega} + \frac{1}{2} e^{-j2\Omega}$$

$$x(n) = \left\{ \underset{\uparrow}{\frac{1}{2}}, 1, \frac{1}{2} \right\}$$

$$\text{Given : } x_p[n] = \begin{cases} x[n], & \text{for } n = 0, 1, 2 \\ 0, & \text{for } n = 3, 4 \end{cases}$$

$$x_p(n) = \left\{ \underset{\uparrow}{\frac{1}{2}}, 1, \frac{1}{2}, 0, 0 \right\} \text{ with period, } N = 5$$

$$a_k = \frac{1}{N} \sum_{n=0}^{N-1} x(n) e^{\frac{-j2\pi}{N} kn}$$

$$a_3 = \frac{1}{5} \sum_{n=0}^4 x(n) e^{\frac{-j6\pi}{5} n}$$

$$|a_3| = 0.038$$

Hence, the correct answer is 0.038.

Method 2

$$\text{We know that, } a_k = \frac{1}{N} \sum_{n=0}^{N-1} x(n) e^{\frac{-j2\pi}{N} kn}$$

$$a_k = \frac{1}{N} X(\Omega) \Big|_{\Omega = \frac{2\pi k}{N}}$$

$$a_k = \frac{1}{N} (1 + \cos \Omega) e^{-j\Omega} \Big|_{\Omega = \frac{2\pi k}{N}} = \frac{1}{N} \left(1 + \cos \frac{2\pi k}{N}\right) e^{\frac{-j2\pi k}{N}}$$

$$a_3 = \frac{1}{5} \left(1 + \cos \frac{2\pi \times 3}{5}\right) e^{\frac{-j2\pi \times 3}{5}}$$

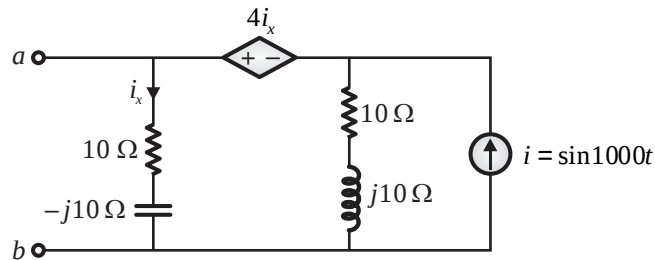
$$a_3 = \frac{1}{5} \left(1 + \cos \frac{6\pi}{5}\right) e^{\frac{-j6\pi}{5}}$$

$$|a_3| = \frac{1}{5} \left(1 + \cos \frac{6\pi}{5}\right) = 0.038$$

Hence, the correct answer is 0.038.

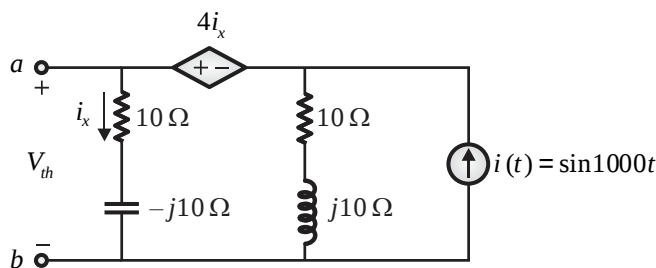
Question 51 Network theory (network theorem)

For the circuit shown, if $i = \sin 1000t$, the instantaneous value of the Thevenin's equivalent voltage (in Volts) across the terminal $a-b$ at time $t = 5$ ms is _____ (Round off to 2 decimal places).

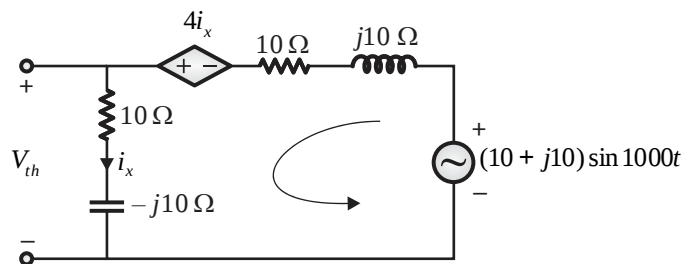


Ans. - 11.98 (- 12.1 to - 11.9)

Sol. Given circuit is shown below,



By source transformation,



$$V_{th} = i_x (10 - j10)$$

By KVL in loop,

$$(10 + j10) \sin 1000t + (10 + j10)i_x - 4i_x + (10 - j10)i_x = 0$$

$$i_x = \frac{(10 + j10) \sin 1000t}{16}$$

$$V_{th} = (10 - j10)i_x$$

$$V_{th} = \frac{(10 - j10)(10 + j10) \sin 1000t}{16}$$

$$V_{th} = \frac{100 + 100}{16} \sin 1000t$$

$$V_{th} = 12.5 \sin 1000t$$

At $t = 5$ msec ,

$$V_{th} = 12.5 \sin 5$$

$$V_{th} = -11.98 \text{ V}$$

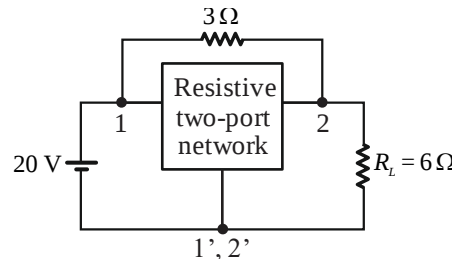
Hence, the correct answer is – 11.98.

Question 52 Network theory (two Port network)

The admittance parameters of the passive resistive two-port network shown in the figure are

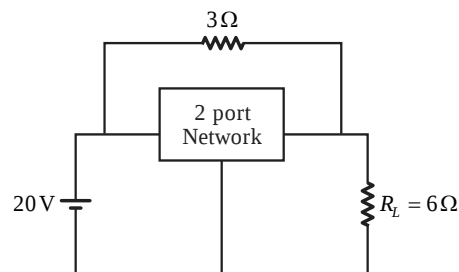
$$y_{11} = 5S, y_{22} = 1S, y_{12} = y_{21} = -2.5S$$

The power delivered to the load resistor R_L in Watt is _____ (Round off to 2 decimal places).



Ans. 237.38 (237 to 239)

Sol. Given two port network and Y-parameters are shown below,



$$Y = \begin{bmatrix} 5 & -2.5 \\ -2.5 & 1 \end{bmatrix} S$$

For 3Ω resistor Y-parameter matrix is,

$$[Y_1] = \begin{bmatrix} \frac{1}{3} & -\frac{1}{3} \\ -\frac{1}{3} & \frac{1}{3} \end{bmatrix} \text{ and given } [Y_2] = \begin{bmatrix} 5 & -2.5 \\ -2.5 & 1 \end{bmatrix}$$

Over all $[Y]$ parameter matrix is,

$$[Y] = \begin{bmatrix} \frac{1}{3} & -\frac{1}{3} \\ -\frac{1}{3} & \frac{1}{3} \end{bmatrix} + \begin{bmatrix} 5 & -2.5 \\ -2.5 & 1 \end{bmatrix}$$

$$[Y] = \begin{bmatrix} \frac{16}{3} & -\frac{8.5}{3} \\ -\frac{8.5}{3} & \frac{4}{3} \end{bmatrix}$$

Y-parameter equation,

$$I_1, I_2 = f(V_1, V_2)$$

$$I_1 = \frac{16}{3}V_1 - \frac{8.5}{3}V_2 \quad \dots\dots(i)$$

$$I_2 = -\frac{8.5}{3}V_1 + \frac{4}{3}V_2 \quad \dots\dots(ii)$$

From the given circuit $V_1 = 20\text{ V}$ for finding V_{th} , output current

$$I_2 = 0$$

$$0 = -\frac{8.5}{3}V_1 + \frac{4}{3}V_2$$

$$\frac{8.5}{3}(20) = \frac{4}{3}V_2$$

$$V_{th} = V_2 = \frac{8.5 \times 20}{4} = 42.5\text{ V}$$

$$R_{th} = \frac{V_2}{I_2} \Big|_{V_1=0\text{ V}}$$

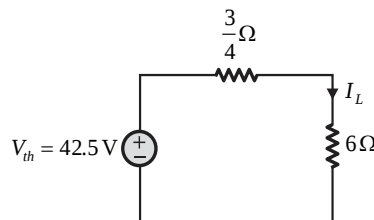
From equation (ii)

$$I_2 = \frac{4}{3}V_2$$

$$R_{th} = \frac{V_2}{I_2}$$

$$R_{th} = \frac{3}{4}\Omega$$

Thevenin's equivalent circuit



$$I_L = \frac{42.5}{\frac{3}{4} + 6}$$

$$I_L = 6.29\text{ A}$$

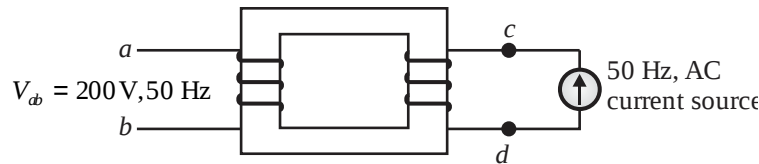
$$P_L = I^2 R_L = (6.29)^2 6$$

$$P_L = 237.38\text{ W}$$

Hence, the correct answer is 237.38.

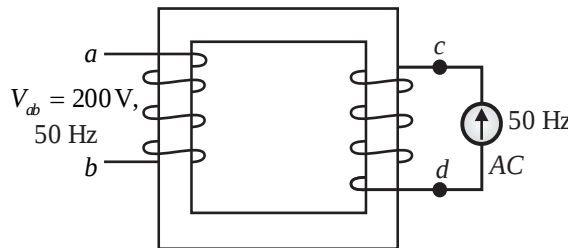
Question 53 Electrical machine (1 phase transformer)

When the winding $c-d$ of the single-phase, 50 Hz, two winding transformer is supplied from an AC current source of frequency 50 Hz, the rated voltage of 200 V (rms), 50 Hz is obtained at the open-circuited terminals $a-b$. The cross sectional area of the core is 5000 mm^2 and the average core length traversed by the mutual flux is 500 mm. The maximum allowable flux density in the core is $B_{\max} = 1 \text{ Wb/m}^2$ and the relative permeability of the core material is 5000. The leakage impedance of the winding $a-b$ and winding $c-d$ at 50 Hz are $(5 + j100\pi \times 0.16) \Omega$ and $(11.25 + j100\pi \times 0.36) \Omega$, respectively. Considering the magnetizing characteristics to be linear and neglecting core loss, the self-inductance of the winding $a-b$ in millihenry is _____. (Round off to 1 decimal place).



Ans. 2200 (2150 to 2250)

Sol. Given : $A = 5000 \text{ mm}^2$, $l = 500 \text{ mm}$, $B_m = 1 \text{ Wb/m}^2$, $\mu_r = 5000$



$$V = E_1 = 200 \text{ V}$$

$$V = 4.44 f \phi_m N_1$$

$$200 = 4.44 \times 50 (1 \times 5000 \times 10^{-6}) N_1$$

$$N_1 = 180.16$$

$$\bar{Z}_{l_1} = (5 + j100\pi \times 0.16) \Omega$$

$$\bar{Z}_{l_2} = (11.25 + j100\pi \times 0.36) \Omega$$

$$L_m = \frac{N^2}{\text{Reluctance}} = \frac{N^2}{\frac{l}{\mu_0 \mu_r A}}$$

$$L_m = \frac{180.16^2}{\frac{500 \times 10^{-3}}{4\pi \times 10^{-7} \times 5000 \times 5000 \times 10^{-6}}} = 2.039 \text{ H or } 2039 \text{ mH}$$

$$L_{l_1} = L_1 - L_m$$

$$L_1 = L_{l_1} + L_m$$

$$L_1 = 0.16 + 2.039$$

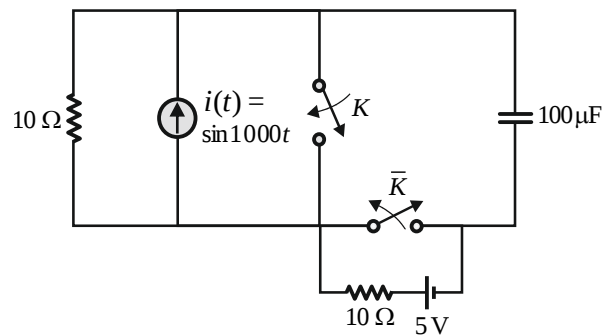
$$L_1 = 2.2\text{H or } 2200\text{ mH}$$

Hence, the self - inductance of winding $a - b$ is 2200 mH.

Hence, the correct answer is 2200.

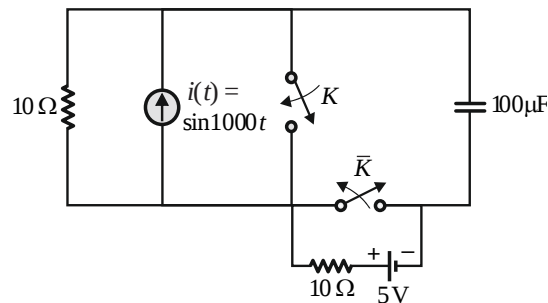
Question 54 Network theory (transient Analysis)

The circuit shown in the figure is initially in the steady state with the switch K in open condition and $\bar{K}$ in closed condition. The switch K is closed and $\bar{K}$ is opened simultaneously at the instant $t = t_1$, where $t_1 > 0$. The minimum value of t_1 in milliseconds, such that there is no transient in the voltage across the $100\mu\text{F}$ capacitor, is _____ (Round off to 2 decimal places).



Ans. 1.57 (1.56 to 1.58)

Sol. Given circuit is as shown below,



Given that K is opened and $\bar{K}$ is closed initially. So given circuit becomes,

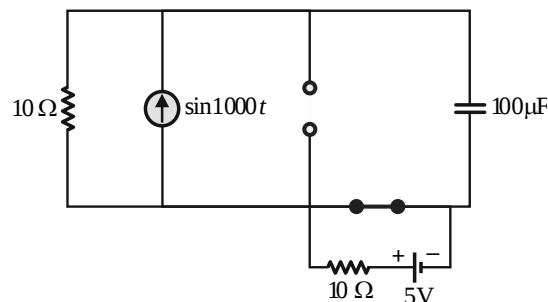
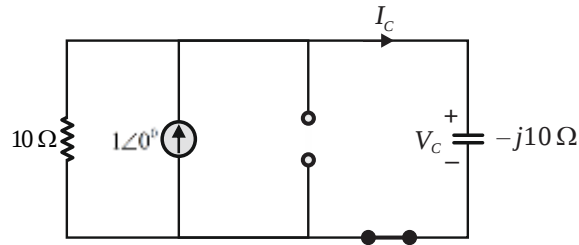


Figure (A) can be drawn in phase form as shown below,

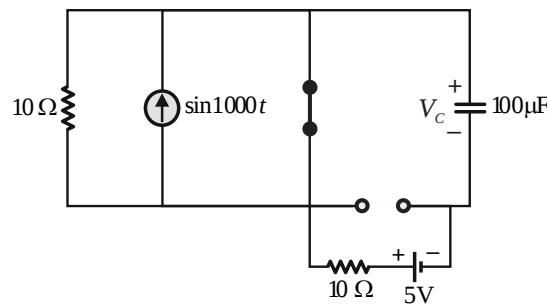


$$\therefore I_C = \frac{1\angle 0^\circ \times 10}{10 - j10} = \frac{1}{\sqrt{2}} \angle 45^\circ$$

$$V_C = -j10 \times I_C = -j10 \times \frac{1}{\sqrt{2}} \angle 45^\circ = \frac{10}{\sqrt{2}} \angle -45^\circ$$

$$\therefore V_C(t) = \frac{10}{\sqrt{2}} \sin(1000t - 45^\circ) \text{ V}$$

Now, at $t = t_1$, K is closed and $\bar{K}$ is opened. So, the circuit becomes,



$\therefore$ Steady state voltage across capacitor will be, $V_C(\infty) = 5$.

Now, for transient free condition at $t = t_1$.

$$V_C(t_1) = V_C(\infty)$$

$$\frac{10}{\sqrt{2}} \sin(1000t_1 - 45^\circ) = 5$$

$$\sin(1000t_1 - 45^\circ) = \frac{5\sqrt{2}}{10} = \frac{1}{\sqrt{2}}$$

$$1000t_1 - 45^\circ = \sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$$

$$1000t_1 - 45^\circ = 45^\circ$$

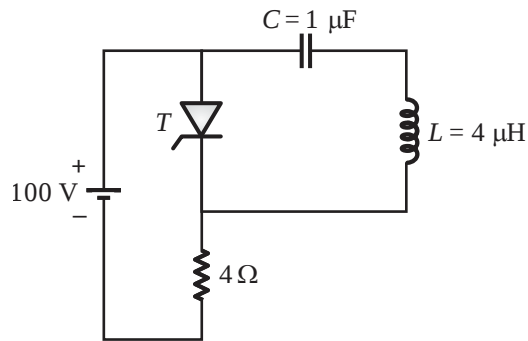
$$1000t_1 = 90^\circ = \frac{\pi}{2}$$

$$t_1 = \frac{\pi}{2} \text{ ms} = 1.57 \text{ ms}$$

Hence, the correct answer is 1.57 ms.

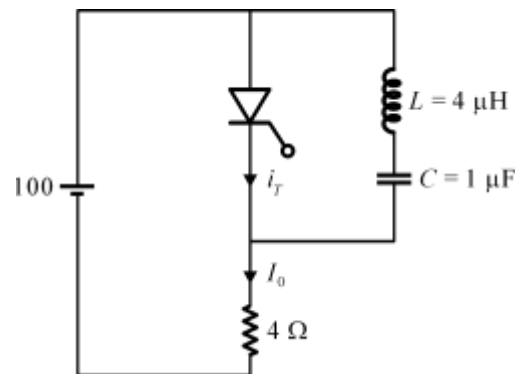
Question 55 Power electronics (power semiconductor device)

The circuit shown in the figure has reached steady state with thyristor 'T' in OFF condition. Assume that the latching and holding currents of the thyristor are zero. The thyristor is turned ON at $t = 0$ sec. The duration in μsec for which the thyristor would conduct, before it turns off, is _____ (round off to 2 decimal places).

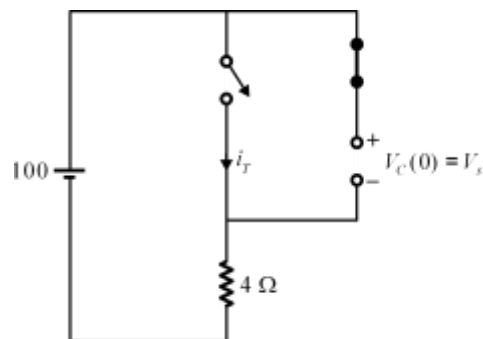


Ans. 7.33 (7.1 to 7.5)

Sol.



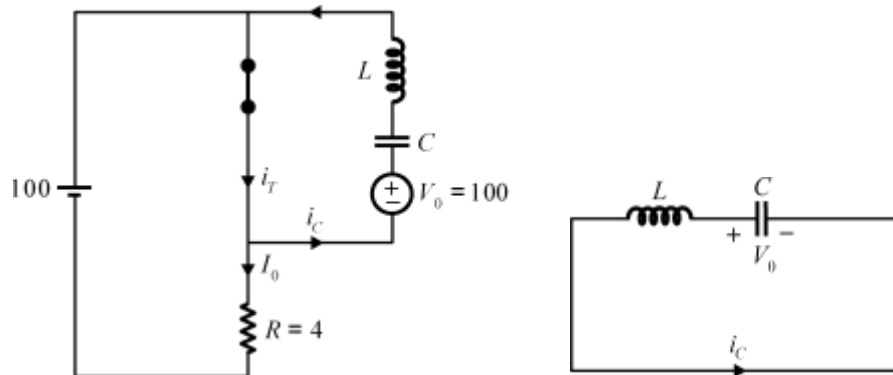
At steady state,



$$V_C(0) = V_s = 100\text{V}$$

$$i_L(0) = 0\text{A}$$

For $t > 0$,



$$i_C = V_s \sqrt{\frac{C}{L}} \sin \omega_0 t = 100 \sqrt{\frac{1}{4}} \sin \omega_0 t$$

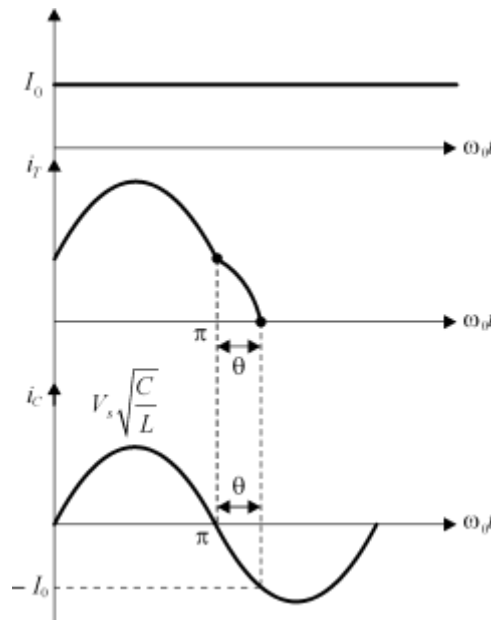
$$i_C = 50 \sin \omega_0 t$$

$$I_0 = \frac{100}{4} = 25 \text{ A}$$

$$\text{Now, } i_T = i_C + I_0$$

$$\text{When } i_C = -i_0$$

$$i_T = 0 \text{ A}$$



$$t_C^* = \frac{\pi}{\omega_0} + \theta$$

$$\theta = \frac{1}{\omega_0} \sin^{-1} \left[\frac{I_0}{I_{cp}} \right]$$

$$\theta = \frac{1}{\omega_0} \times \sin^{-1} \left[\frac{25}{50} \right]$$

$$\theta = \frac{\pi}{6\omega_0}$$

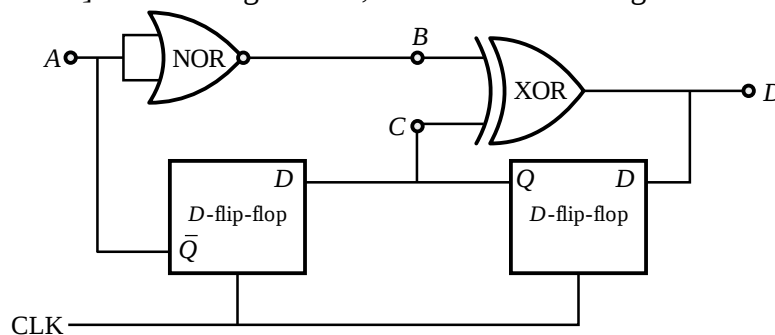
$$t_C^* = \frac{\pi}{\omega_0} + \frac{\pi}{6\omega_0} = \frac{1}{\omega_0} \left[\frac{7\pi}{6} \right] = \frac{7\pi}{6} \times \sqrt{LC}$$

$$t_C^* = 7.33 \mu\text{sec}$$

Hence, the correct answer is 7.33.

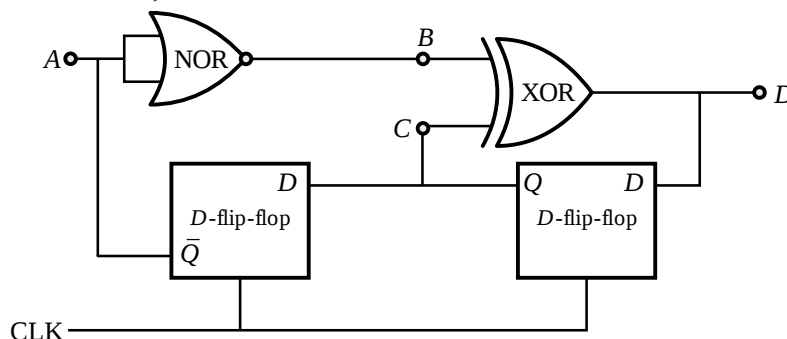
Question 56 Digital Electronics (logic gate)

Neglecting the delays due to the logic gates in the circuit shown in figure, the decimal equivalent of the binary sequence $[ABCD]$ of initial logic states, which will not change with clock, is _____.



Ans. 8 (8 to 8)

Sol. Given circuit is shown below,



$Q_L \ Q_R$	$A = \bar{Q}_L$	$B = \bar{A}$	$C = Q_R$	$D = B \oplus C$	$D_L = Q_R$	$D_R = D$
00	1	0	0	0	0	0

$\therefore$ Initial state does not change when $ABCD = 1000 = (8)_{10}$

Hence, the correct answer is 8.

Question 57 Microprocessor

In a given 8-bit general purpose micro-controller there are following flags.

C-Carry, A-Auxiliary Carry, O-Overflow flag, P-Parity (0 for even, 1 for odd)

R0 and R1 are the two general purpose registers of the micro-controller.

After execution of the following instructions, the decimal equivalent of the binary sequence of the flag pattern $[CAOP]$ will be _____.

MOV R0, +0x60

MOV R1, +0x46

ADD R0, R1

Ans. 2 (2 to 2)**Sol.** $R0 \leftarrow 60H$ $R1 \leftarrow 46H$ ADD R0, R1 $\Rightarrow$ 0110 0000

+ 0100 0110

1010 0110 $\therefore$ Carry flag = 0

Over flow flag = 1

Auxiliary carry flag = 0

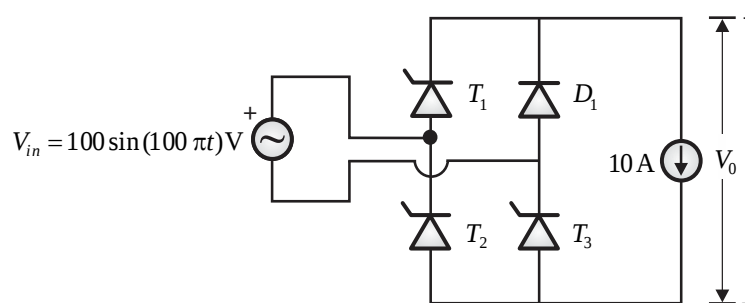
Parity flag = 0

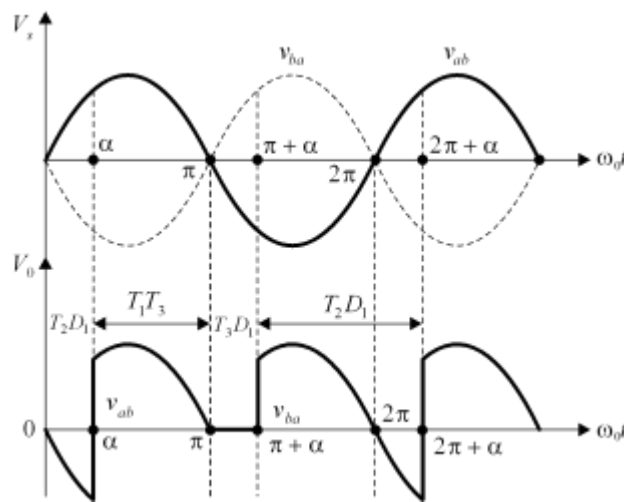
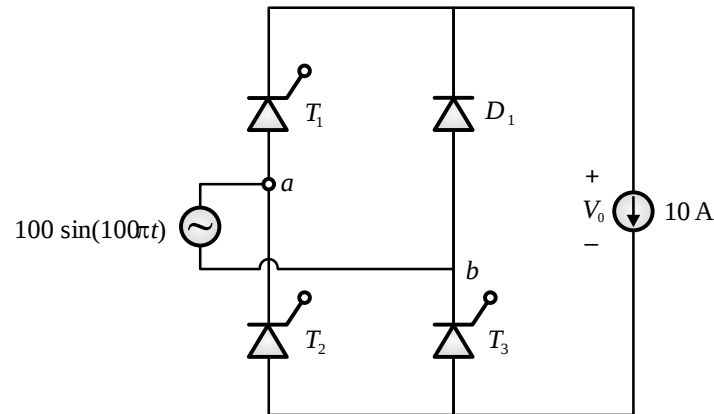
CAOP = $(0010)_2 = (2)_{10}$

Hence, the correct answer is 2.

Question 58 Power electronics (single phase rectifier)

The single phase rectifier consisting of three thyristors T_1, T_2, T_3 and a diode D_1 feed power to a 10 A constant current load. T_1 and T_3 are fired at angle $\alpha = 60^\circ$ and T_2 is fired at $\alpha = 240^\circ$. The reference for α is positive zero crossing of V_{in} . The average voltage V_0 across the load in volts is _____. (round off to 2 decimal places).

**Ans. 39.79 (39 to 40.5)****Sol.**



$(\alpha) - (\pi) \rightarrow T_1 T_3$ conduct $\rightarrow v_0 = v_{ab} = V_m \sin \omega t$

$(\pi) - (\pi + \alpha) \rightarrow T_3 D_1$ conduct $\rightarrow v_0 = 0$

$(\pi + \alpha) - (2\pi + \alpha) \rightarrow T_2 D_1$ conduct $\rightarrow v_0 = -V_m \sin \omega t$

$$V_{0(avg)} = \frac{1}{2\pi} \left[\int_{\alpha}^{\pi} V_m \sin \omega t d\omega t - \int_{\pi+\alpha}^{2\pi+\alpha} V_m \sin(\omega t) d\omega t \right]$$

$$= \frac{V_m}{2\pi} \left[-\cos \omega t \Big|_{\alpha}^{\pi} - \left(-\cos \omega t \Big|_{\pi+\alpha}^{2\pi+\alpha} \right) \right]$$

$$= \frac{V_m}{2\pi} \left[(1 + \cos \alpha) - [\cos(\pi + \alpha) - \cos(2\pi + \alpha)] \right]$$

$$= \frac{V_m}{2\pi} [(1 + \cos \alpha) - (-\cos \alpha - \cos \alpha)]$$

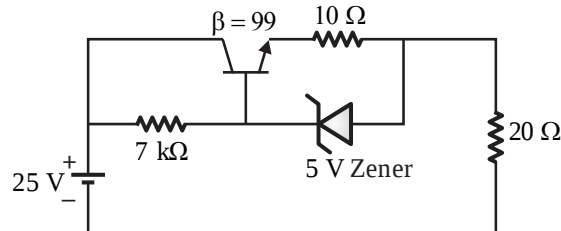
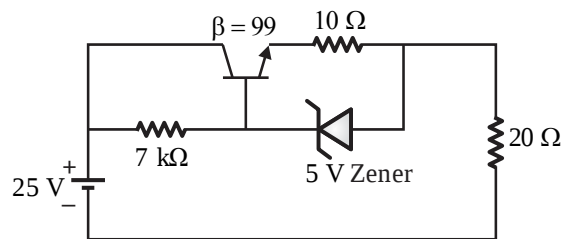
$$= \frac{V_m}{2\pi} [1 + 3\cos \alpha] = \frac{100}{2\pi} [1 + 3\cos 60^\circ]$$

$$V_{0(avg)} = \frac{250}{2\pi} = 39.789 \text{ V}$$

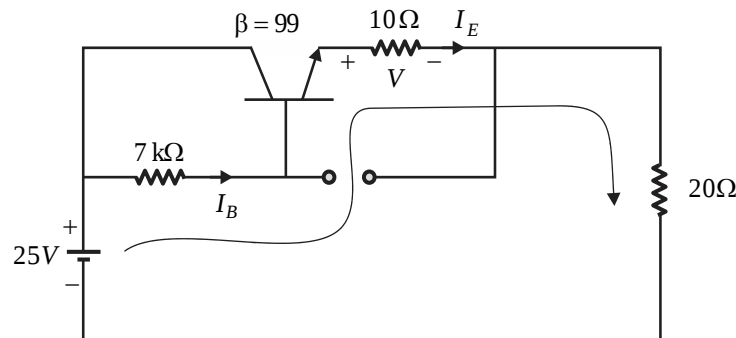
Hence, the correct answer is 39.79

Question 59 Analog Electronics (zener diode regulator circuit)

The Zener diode in circuit has a breakdown voltage of 5 V. The current gain β of the transistor in the active region is 99. Ignore base-emitter voltage drop V_{BE} . The current through the $20\ \Omega$ resistance in milliamperes is _____. (rounded off to 2 decimal places).

**Ans. 250 (245 to 255)****Sol.** Given circuit is shown below,

Assume that the zener diode is OFF.



Applying KVL in loop shown,

$$25 - I_B \times 7\text{ k}\Omega - I_E \times (10\ \Omega + 20\ \Omega) = 0$$

$$25 - I_B \times 7\text{ k}\Omega - (\beta + 1)I_B \times 30\ \Omega = 0$$

$$25 - I_B \times 7\text{ k}\Omega - 100I_B \times 30\ \Omega = 0$$

$$I_B = \frac{25}{10000} = 2.5\text{ mA}$$

$$\therefore I_E = 100 \times 2.5 = 250\text{ mA}$$

$$V = 250\text{ mA} \times 10 = 2.5\text{ V} < 5\text{ V}$$

$\therefore$ Zener diode is not in breakdown and assumption is correct.

Hence, the correct answer is 250.

Question 60 Power system (per unit system)

The two-bus power system shown in figure (i) has one alternator supplying a synchronous motor load through a Y-Δ transformer. The positive, negative and zero-sequence diagrams of the system are shown in figures (ii), (iii) and (iv), respectively. All reactances in the sequence diagrams are in p.u. For a bolted line-to-line fault (fault impedance = zero) between phases 'b' and 'c' at bus 1, neglecting all pre-fault currents, the magnitude of the fault current (from phase 'b' to 'c') in p.u. is _____. (Round off to 2 decimal places).

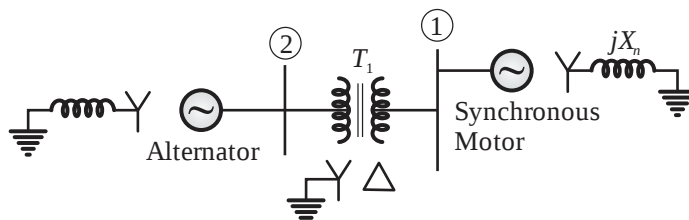


Figure (i): Single-line diagram of the power system

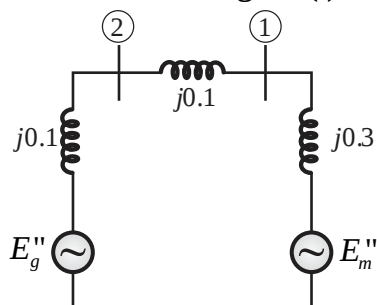


Figure (ii): Positive-sequence network

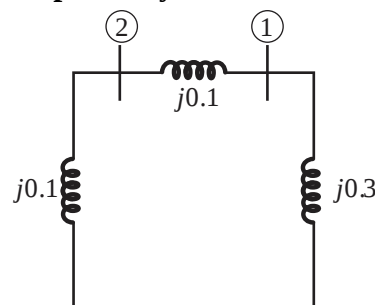


Figure (iii): Negative-sequence network

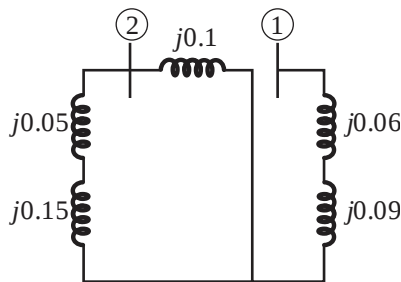
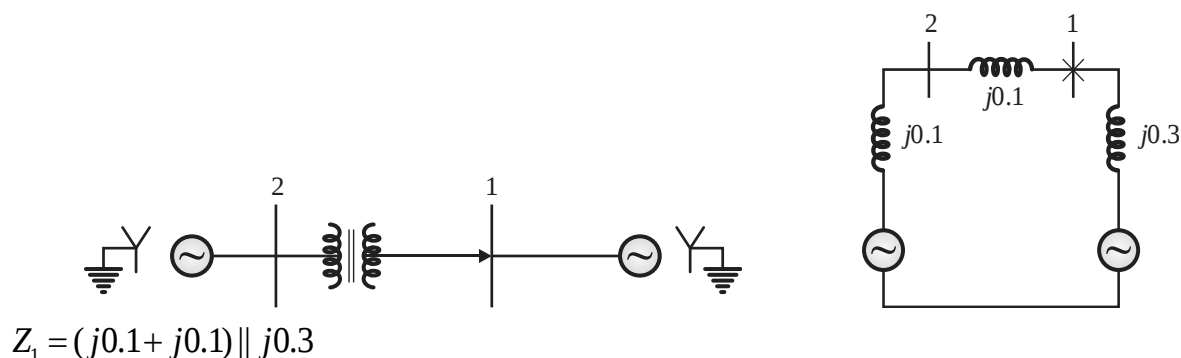


Figure (iv): Zero-sequence network

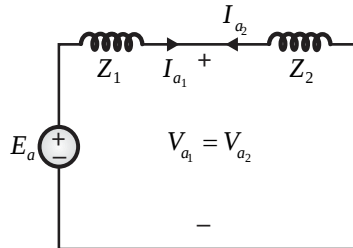
Ans. 7.22 (7.1 to 7.3)

Sol.



$$Z_1 = j0.12$$

$$Z_1 = Z_2 = j0.12$$



$$I_{a_1} = \frac{E_a}{Z_1 + Z_2} = \frac{1 \angle 0}{j0.12 + j0.12}$$

$$I_{a_1} = 4.167 \angle -90^\circ$$

$$I_{a_2} = -I_{a_1} = 4.167 \angle 90^\circ$$

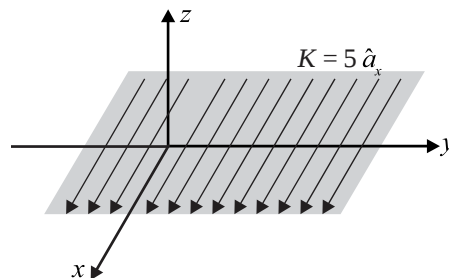
$$I_b = I_{b_0} + I_{b_1} + I_{b_2} = (I_{a_0} = 0) + \alpha^2 I_{a_1} + \alpha I_{a_2}$$

$$I_f = I_b = (\alpha^2 - \alpha) I_{a_1} = 7.22 \angle 180^\circ \text{ pu}$$

Hence, the correct answer is 7.22.

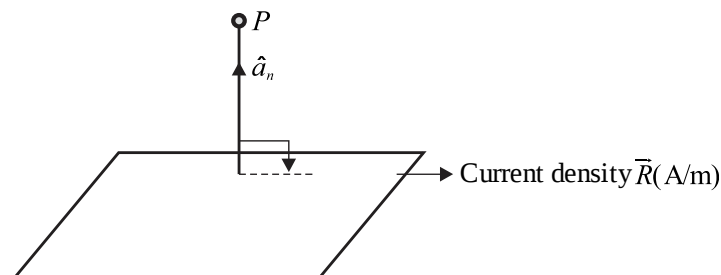
Question 61 Electromagnetic field (magnetostatic)

An infinite surface of linear current density $K = 5 \hat{a}_x$ A/m exists on the x - y plane, as shown in the figure. The magnitude of the magnetic field intensity (H) at a point (1, 1, 1) due to the surface current in A/m is _____ (Round off to 2 decimal places).



Ans. 2.5 (2.49 to 2.51)

Sol. $\vec{H}_p = \frac{1}{2} (\vec{K} \times \hat{a}_n)$

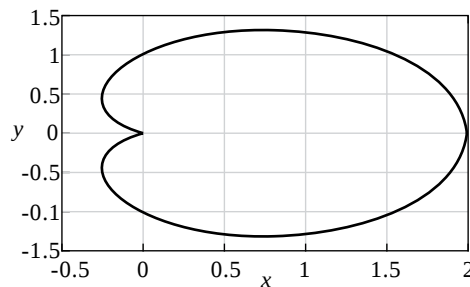


$$\vec{H}_p = \frac{1}{2}(5\hat{a}_x \times \hat{a}_z) = -2.5\hat{a}_y$$

$$|\vec{H}_p| = 2.5 \text{ A/m}$$

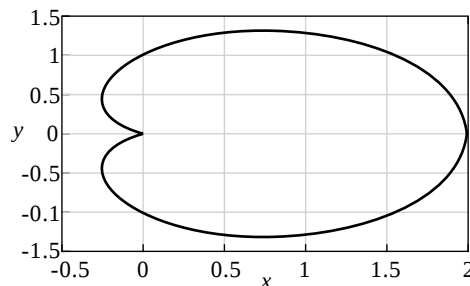
Question 62 Electromagnetic (electrostatic)

The closed curve shown in the figure is described by $r=1+\cos\theta$, where $r=\sqrt{x^2+y^2}$; $x=r\cos\theta$, $y=r\sin\theta$. The magnitude of the line integral of the vector field $\vec{F} = -y\hat{i} + x\hat{j}$ around the closed curve is _____. (round off to two decimal places)



Ans. 9.42 (9 to 10)

Sol. Given closed curve,



Given vector is, $\vec{F} = -y\hat{i} + x\hat{j}$

$$\oint \vec{F} \cdot d\vec{l} = \oint (-y)dx + (x)dy$$

$$\oint \vec{F} \cdot d\vec{l} = \iint 1 - (-1)dx dy = 2 \iint dx dy = 2 (\text{Area of the cardioid } r = 1 + \cos\theta) = 2 \left\{ \int_0^{2\pi} \int_0^{1+\cos\theta} dr d\theta \right\}$$

$$= 2 \left\{ \int_0^{2\pi} \frac{(1+\cos\theta)^2}{2} d\theta \right\} = \left\{ \int_0^{2\pi} \left(2\cos^2 \frac{\theta}{2} \right) d\theta \right\}$$

$$= \int_0^{2\pi} \left(4\cos^4 \frac{\theta}{2} \right) d\theta$$

$$\frac{\theta}{2} = t$$

$$\frac{d\theta}{2} = dt, \text{ for } \theta = 2\pi \quad t = \pi, \text{ for } \theta = 0 \quad t = 0$$

$$= \int_0^{\pi} 4 \cos^4 t \cdot 2 dt = 8 \int_0^{\pi} \cos^4 t dt$$

$$= 8(2) \int_0^{\pi/2} \cos^4 t dt = 16 \left(\frac{3.1}{4.2} \cdot \frac{\pi}{2} \right) = 3\pi = 9.42$$

Hence, the correct answer is 9.42

Question 63 Signals & Systems (sampling)

A signal $x(t) = 2 \cos(180\pi t) \cos(60\pi t)$ is sampled at 200 Hz and then passed through an ideal low pass filter having cut off frequency of 100 Hz. The maximum frequency present in the filtered signal in Hz is _____ (round off to the nearest integer).

Ans. 80 (80 to 80)

Sol. Given that, $x(t) = 2 \cos(180\pi t) \cos(60\pi t) = \cos(240\pi t) + \cos(120\pi t)$

Where, $f_1 = 120$ Hz, $f_2 = 60$ Hz

Frequency components present at sampler output,

$$\Rightarrow f_1, f_s \pm f_1, 2f_s \pm f_1, \dots$$

$$f_2, f_s \pm f_2, 2f_s \pm f_2, \dots$$

$$\Rightarrow 120, 200 \pm 120, \dots$$

$$60, 200 \pm 60, \dots$$

$$\Rightarrow 120, 80, 320, \dots$$

$$60, 140, 260, \dots$$

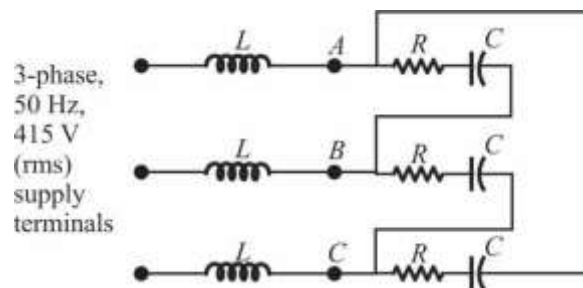
Now, LPF will pass only 60 Hz and 80 Hz.

So, maximum frequency available at LPF output is 80 Hz.

Hence, the correct answer is 80.

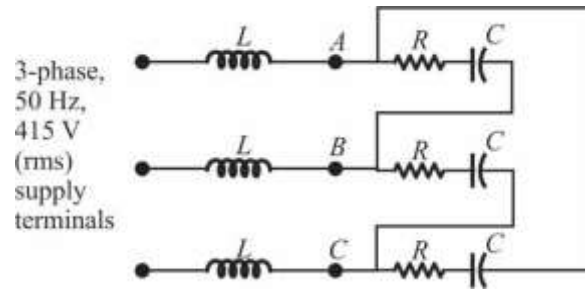
Question 64 Network theory (3 phase circuit)

A balanced delta connected load consisting of the series connection of one resistor ($R = 15\Omega$) and a capacitor ($C = 212.21 \mu\text{F}$) in each phase is connected to 3-phase, 50 Hz, 415 V supply terminals through a line having an inductance of $L = 31.83 \text{ mH}$ per phase, as shown in figure. Considering the change in the supply terminal voltage with loading to be negligible, the magnitude of the voltage across the terminals V_{AB} in volts is _____ (rounded off to nearest integer).



Ans. 415 (414 to 416)

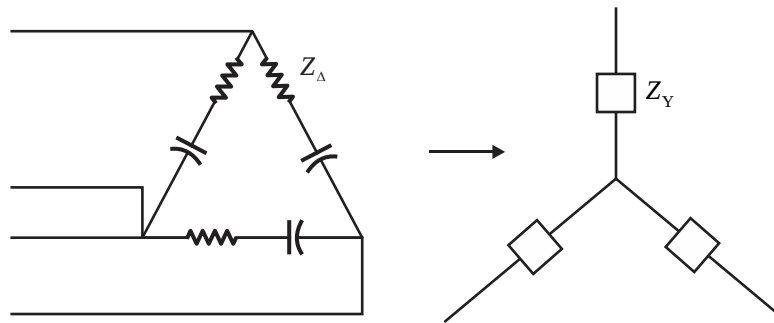
Sol.



$$X_L = 2\pi \times 50 \times 31.83 \times 10^{-3} = 10 \Omega$$

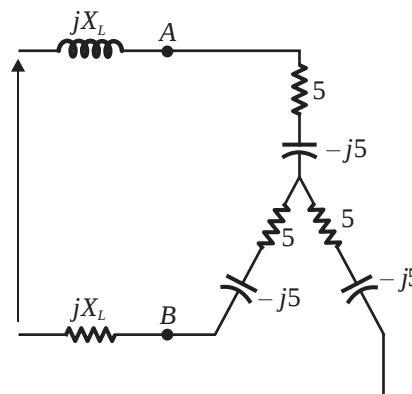
$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 50 \times 212.21 \times 10^{-6}} = 15$$

$$\bar{Z} = R - jX_C = 15 - j15$$



$$Z_Y = \frac{Z_\Delta}{3} = 5 - j5$$

$$V_p = \frac{415}{\sqrt{3}}$$



$$\bar{V}_{AN} = \frac{415}{\sqrt{3}} \angle 0^\circ \times \frac{5 - j5}{5 - j5 + j10} = 240 \angle -90^\circ$$

$$\bar{V}_{BN} = \frac{415}{\sqrt{3}} \angle -120^\circ \times \frac{5 - j5}{5 - j5 + j10} = 240 \angle 150^\circ$$

$$\bar{V}_{AB} = \bar{V}_{AN} - \bar{V}_{BN}$$

$$\bar{V}_{AB} = 240 \angle -90^\circ - (240 \angle 150^\circ) = 415 \angle -60^\circ$$

$$|V_{AB}| = 415 \text{ V}$$

Question 65 Mathematics (Vector Calculus)

A quadratic function of two variables is given as $f(x_1, x_2) = x_1^2 + 2x_2^2 + 3x_1 + 3x_2 + x_1x_2 + 1$.

The magnitude of maximum rate of change of the function at the point (1,1) is _____. (Rounded off to the nearest integer)

Ans. 10 (10 to 10)

Sol. Given : $f(x_1, x_2) = x_1^2 + 2x_2^2 + 3x_1 + 3x_2 + x_1x_2 + 1$

$$\nabla f = \hat{i} \frac{\partial f}{\partial x_1} + \hat{j} \frac{\partial f}{\partial x_2} = \hat{i}(2x_1 + 3 + x_2) + \hat{j}(4x_2 + 3 + x_1)$$

$$\nabla f_{(1,1)} = 6\hat{i} + 8\hat{j}$$

$$|\nabla f_{(1,1)}| = \sqrt{6^2 + 8^2} = \sqrt{36 + 64} = \sqrt{100} = 10$$

Hence, the correct answer is 10.

□□□