General Aptitude

Q.1 to Q.5 Carry ONE Mark Each

Question 1

If '$\rightarrow$' denotes increasing order of intensity, then the meaning of the words [talk $\rightarrow$ shout $\rightarrow$ scream] is analogous to [please $\rightarrow$ ______ $\rightarrow$ pander].

Which one of the given options is appropriate to fill the blank?

(A) flatter
(B) flutter
(C) fritter
(D) frizzle

Ans. **(A)**

Sol. **Given :** If $\rightarrow$denotes increasing order of intensity.

[Talk $\rightarrow$ Shout $\rightarrow$ Scream]

From option (A) :

Flatter : To praise someone in order to make them feel attractive or important. Sometime in a way that is not sincere.

NOTE : Pander and flatter are semantically related in same cases you can use pander instead a verb flatter.

Hence option (A) is correct

From option (B) :

Flutter : To move or make something move quickly and lightly especially through the air

Hence, option (B) is Incorrect

From option (C)

Fritter : Can be both a Noun and a verb. As a Noun it refers to a small piece of food. Such as fruit meat or vegetables that is coated in batter and fried until crisp. As a verb. It means to waste time, money or energy on trifling matters or activities that have little or no value.

Hence option (C) is incorrect

From option (D) :

Frizzle : To become or make something crisp (Pleasantly dry and hard) or slightly burned by frying or grilling.

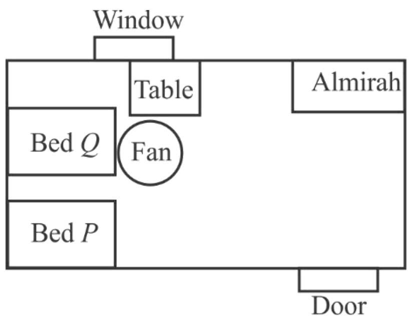
Hence option (D) is incorrect.

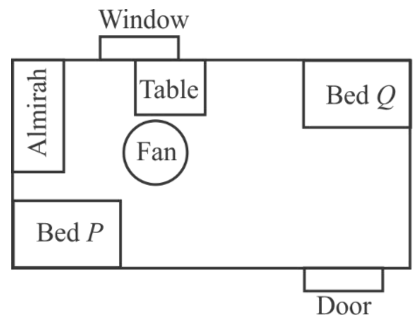
Question 2

P and *Q* have been allotted a hostel room with two beds, a study table, and an almirah. *P* is an avid bird-watcher and wants to sit at the table and watch birds outside the window. *Q* does not mind that as long as his bed is close to the ceiling fan.

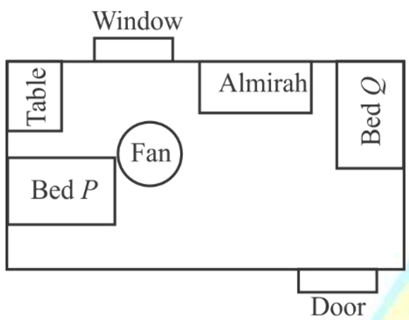
Which one of the following arrangements suits them the most?

(A)
(B)

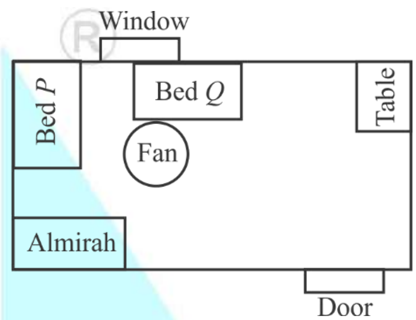




(C)



(D)



Ans. (A)

Sol. Given :

(i) Two beds

(ii) A study table

(iii) An almirah

- "P" → an avid bird watcher
  → wants to sit at the table
  → watch birds outside the window
- Q does not mind that as long as his bed is close to the ceiling fan.

From, the given explanation (above), option (A) arrangement suits them the most.

Hence the correct option is (A).

Question 3

The decimal number system uses the characters 0, 1, 2, …., 8, 9, and the octal number system uses the characters 0, 1, 2, …., 6, 7.

For example, the decimal number 12 $(=1\times10^1+2\times10^0)$ is expressed as 14 $(=1\times8^1+4\times8^0)$ in the octal number system.

The decimal number 108 in the octal number system is

(A) 168 (B) 108

(C) 150 (D) 154

Ans. (D)

Sol. **Method 1**

Decimal to octal conversion using by Direct method :

To Convert a Decimal Number to an octal Number directly you must start dividing the Number by 8 unit you get 0 as the quotient. This is a straight forward method that Involves dividing the Number to be Converted

Step 1 : If decimal Number is N. Divide it by 8. Because the octal Number Systems base is 8.

Step 2 : Note the value of the residual which will be one of the flowing: 0, 1, 2, 3, 4, 5, 6 or 7. Divide the remaining decimal number unit it Equals 0 and record the remainder of each step.

Step 3 : Then from bottom to top write the remainder which will be the Equivalent octal number of the provided decimal Number.

Given : Decimal Number = 108

8	108		MSD
8	13	4	↑
8	1	5	↑
	0	1	LSD

$$(108)_{10} = (154)_8$$

Method 2

Convert Decimal to Binary to octal :

Step 1 : Divide the Number by 2

Step 2 : Get the Integer quotient for Next Iteration.

Step 3 : Get the remainder for the Binary digit

Step 4 : Repeat the above steps unitl the quotient is Equal to 0.

Given : Decimal Number = 108

2	108		MSD
2	54	0	↑
2	27	0	↑
2	13	1	↑
2	6	1	↑
2	3	0	↑
2	1	1	↑
		1	LSD

$$(108)_{10} = (1101100)_1$$

Now Convert Binary number into an octal Number

0 0 1	1 0 1	1 0 0
1	5	4

$$(108)_{10} = (154)_8$$

Hence the correct option is (D).

Question 4

A shopkeeper buys shirts from a producer and sells them at 20% profit. A customer has to pay Rs. 3,186.00 including 18% taxes, per shirt. At what price did the shopkeeper buy each shirt from the producer ?

(A) Rs. 2,500.00

(B) Rs. 1,975.40

(C) Rs. 2,250.00

(D) Rs, 2,548.80

Ans. **(C)**

Sol. As we know that :

$$\text{Profit} = \text{S.P} - \text{C.P}$$

$$\%\ \text{Profit} = \frac{\text{S.P} - \text{C.P}}{\text{C.P}}$$

Given : Profit (%) $= 20\%$

Let : $\text{C.P} = x$

$$0.2 = \frac{\text{S.P}}{\text{C.P}} - 1$$

$$\frac{\text{S.P}}{\text{C.P}} = 1.2$$

where, SP = Selling Price

CP = Cost Price

$$\frac{3186}{x + \frac{18x}{100}} = 1.2$$

$$\frac{3186}{1.18x} = 1.2$$

$$x = \frac{3186}{1.18 \times 1.2}$$

$$x = 2250$$

Hence the correct option is (C).

Question 5

If, for non - zero real variables x, y and real parameter $a > 1$,

$$x : y = (a+1) : (a-1)$$

Then, the ratio $(x^2 - y^2) : (x^2 + y^2)$ is

(A) $2a : (a^2 + 1)$

(B) $a : (a^2 + 1)$

(C) $2a : (a^2 - 1)$

(D) $a:(a^2-1)$

Ans. **(A)**

Sol. Given : If for Non-zero real variable x, y and real parameter a >1,

$$x:y=(a+1):(a-1)$$

$$\frac{x}{y}=\frac{(a+1)}{(a-1)}$$

$$\frac{x^2-y^2}{x^2+y^2}=\frac{\left(\frac{x}{y}\right)^2-1}{\left(\frac{x}{y}\right)^2+1}$$

$$\frac{x^2-y^2}{x^2+y^2}=\frac{\left[\left(\frac{a+1}{a-1}\right)^2-1\right]}{\left[\left(\frac{a+1}{a+1}\right)^2+1\right]}$$

$$=\frac{(a-1)^2-(a-1)^2}{(a+1)^2+(a-1)^2}$$

$$\frac{x^2-y^2}{x^2+y^2}=\frac{4a}{2a^2+2}=\frac{2a}{a^2+1}$$

Hence the correct option is (A)

Q.6 to Q.10 Carry TWO Marks Each

Question 6

In the given text, the blanks are numbered (i) – (iv). Select the best match for all the blanks.

Following a row __(i)__ the shopkeeper __(ii)__ the price of a frying pan, the cook stood __(iii)__ a row to withdrawn cash ___(iv)___the ATM booth.

(A) (i) with (ii) over (iii) at (iv) with

(B) (i) at (ii) over (iii) over (iv) in

(C) (i) with (ii) over (iii) in (iv) at

(D) (i) over (ii) with (iii) over (iv) at

Ans. **(C)**

Sol. The prepositions in option (C) accurately represent the relationships between the actions and locations in the context.

Hence the correct option is (C).

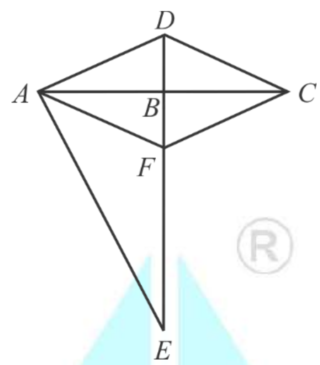
Question 7

In the following figure.

$CD = 5$ cm. $BE = 10$ cm, $AE = 12$ cm.

$\angle DAB = \angle DCB$, and $\angle DAE = \angle DBC = 90^0$

Points $AFCD$ create a rhombus.



The length of BF (in cm) is

(A) 3 (B) 2

(C) 4 (D) 6

Ans. **MTA**

Sol. Given figure is represent the geometry similar to the Rhombus.

Note :

1. Diagonals of a rohmbus intersect each other at 90^0.

2. All sides of a rohmbus are equal in length.

3. Adjacent angles add up to 180^0.

So, in a rhombus → from figure.

$$CD = AD = AF = FC$$

Given :

(i) $CD = 5$ cm

(ii) $AE = 12$ cm

(iii) $BE = 10$ cm

then $CD = AD = AF = FC = 5$ cm

(iv) $\angle DAB = \angle DCB$ and

$\angle DAE = \angle DBC = 90^0$

then $\angle DBC = \angle DBA = \angle ABF = \angle FBC = 90^0$

In $\triangle DAE$:

$$(DE)^2 = (DA)^2 + (AE)^2$$

$$DE = \sqrt{(5)^2 + (12)^2} = 13 \text{ cm}$$

- $DE = DB + BE \Rightarrow DB = DE - BE$

$$DB = 13 - 10 = 3 \text{ cm}$$

- $DB = 3 \text{ cm} = BF$

$$DB = BF$$

From property –1 above.

$$DB = BF = 3 \text{ cm}$$

Statement - I : In $\triangle ABF$

$$(AF)^2 = (AB)^2 + (BF)^2$$

$$AB = \sqrt{(AF)^2 - (BF)^2}$$

$$AB = \sqrt{(5)^2 - (3)^2} = 4 \text{ cm}$$

$\because$ $AF = CD = 5$ cm

$$AB = 4 \text{ cm}$$

Statement - II : In $\triangle ABE$ $(= 90^0)$

$$(AE)^2 = (AB)^2 + (BE)^2$$

$$AB = \sqrt{(AE)^2 - (BE)^2}$$

$$AB = \sqrt{(12)^2 - (10)^2} = \sqrt{44} \text{ cm}$$

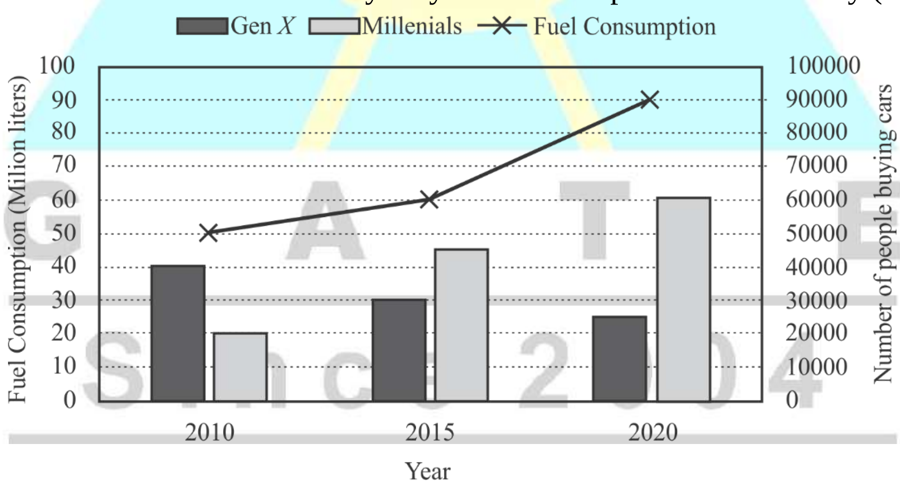
For the same figure value of "AB" is came different.

$\because$ $AB = 4 \text{ cm} \neq AB = \sqrt{44}$ cm

So, this reasons the given data and given figure did not match properly hence M.T.A.

Question 8

The chart below shows the data of the number of cars bought by Millennials and Gen C people in a country from the year 2010 to 2020 as well as the yearly fuel consumption of the country (in Million liters).



Considering the data presented in the chart, which one of the following options is true?

(A) The percentage increase in fuel consumption from 2010 to 2015 is more than the percentage increase in fuel consumption from 2015 to 2020.

(B) The increase in the number of Millennial car buyers from 2015 to 2020 is less than the decrease in the number of Gen X car buyers from 2010 to 2015.

(C) The increase in the number of Millennial car buyers from 2010 to 2015 is more than the decrease in the number of Gen X car buyers from 2010 to 2015.

(D) The decrease in the number of Gen X car buyers from 2015 to 2020 is more than the increase in the number of Millennial car buyers from 2010 to 2015.

Ans. **(C)**

Sol. (i) The increase in the number of Millennial car buyers from 2010 to 2015

$= 45000 - 20000 = 25000$

(ii) The decrease in the number of Gen X Car buyers from 2010 to 2015

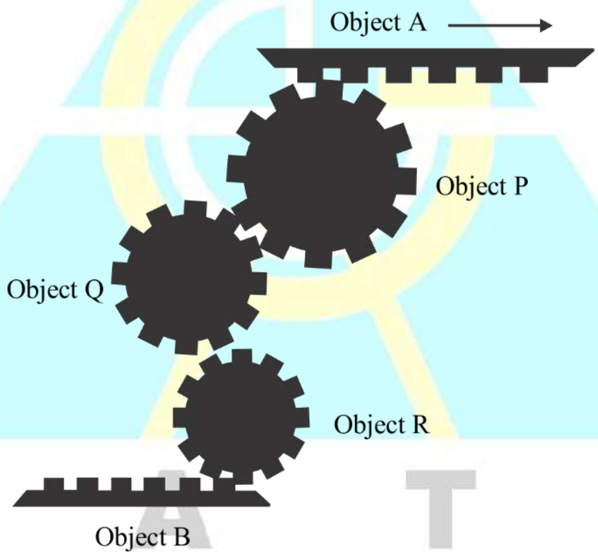
$= 40000 - 30000 = 10000$

So, the increase in the number of millennial car buyers from 2010 to 2015 is more than the decrease in the number of Gen X Car buyers from 2010 to 2015.

Hence, the correct option is (C).

Question 9

The assembly shown below has three teethed circular objects (Pinions) and two teethed flat objects (Racks), which are perfectly mating with each other. Pinions can only rotate clockwise or anti-clockwise staying at its own center. Racks can translate towards the left $(\leftarrow)$ or the right $(\rightarrow)$ direction.



If the object A (Rack) is translating towards the right $(\rightarrow)$ direction, the correct statement among the following is

(A) Object B translates towards the right direction.

(B) Object B translates towards the left direction.

(C) Object R rotates in the anticlockwise direction.

(D) Object Q rotates in the clockwise direction.

Ans. **(B)**

Sol. Object B translate towards the left direction

Hence the correct option is (B).

Question 10

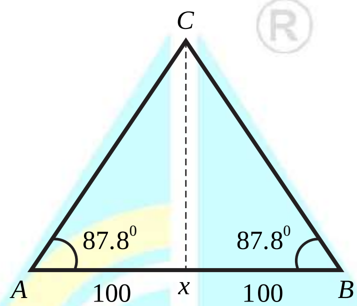
A surveyor has to measure the horizontal distance from her position to a distant reference point C. Using her position as the center, a 200 m horizontal line segment is drawn with the two endpoints A and B. Points A, B and C are not collinear. Each of the angles $\angle CAB$ and $\angle CBA$ are measured as 87.8^0. The distance (in m) of the reference point C from her position is nearest to

(A) 2603 (B) 2606

(C) 2306 (D) 2063

Ans. (A)

Sol. According to given data:



$$\tan\theta = \frac{cx}{Ax}$$

$$\tan(87.8^0) = \frac{cx}{100}$$

$$cx = 100 \times \tan(87.8^0)$$

$$cx = 2603$$

Hence the correct option is (A).

Technical Section

Q.11 to Q.35 Carry ONE Mark Each

Question 11 ***[Mathematics : Linear Algebra]***

Which one of the following matrices has an inverse ? **(1 Mark)**

(A) $\begin{bmatrix} 1 & 4 & 8 \\ 0 & 4 & 2 \\ 0.5 & 2 & 4 \end{bmatrix}$ (B) $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 2 & 9 \end{bmatrix}$

(C) $\begin{bmatrix} 1 & 4 & 8 \\ 0 & 4 & 2 \\ 1 & 2 & 4 \end{bmatrix}$ (D) $\begin{bmatrix} 1 & 4 & 8 \\ 0 & 4 & 2 \\ 3 & 12 & 24 \end{bmatrix}$

Ans. (C)

Sol. The inverse of a non-singular square matrix is given by.

$$A^{-1} = \frac{Adj(A)}{|A|} \qquad \left[|A| \neq 0\right]$$

Checking from the options :

From option (A) :

$$A=\begin{bmatrix}1 & 4 & 8\\0 & 4 & 2\\0.5 & 2 & 4\end{bmatrix}$$

Applying elementary transformation

$R_1 \to R_1 - 2R_3$

$$A=\begin{bmatrix}0 & 0 & 0\\0 & 4 & 2\\0.5 & 2 & 4\end{bmatrix}$$

Determinant of $A = 0$

Hence, A is singular matrix and it's inverse does not exist.

From option (B) :

$$A=\begin{bmatrix}1 & 2 & 3\\2 & 4 & 6\\3 & 2 & 9\end{bmatrix}$$

Applying elementary transformation

$R_2 \to R_2 - 2R_1$

$$A=\begin{bmatrix}1 & 2 & 3\\0 & 0 & 0\\3 & 2 & 9\end{bmatrix}$$

Determinant of $A = 0$

From option (C) :

$$A=\begin{bmatrix}1 & 4 & 8\\0 & 4 & 2\\1 & 2 & 4\end{bmatrix}$$

Determinant $=1(16-4)-4(0-2)+8[0-4]$

$$=12+8-32$$

$$=(-12)\neq 0$$

Hence, A is non singular matrix and it's inverse will exist.

From option (D) :

$$A=\begin{bmatrix}1 & 4 & 8\\0 & 4 & 2\\3 & 12 & 24\end{bmatrix}$$

Applying elementary transformation

$R_3 \rightarrow R_3 - 3R_1$

$$A=\begin{bmatrix}1 & 4 & 8\\0 & 4 & 2\\0 & 0 & 0\end{bmatrix}$$

Determinant of $A = 0$

Hence, A is singular matrix and it's inverse does not exist.

Hence the correct option is (C).

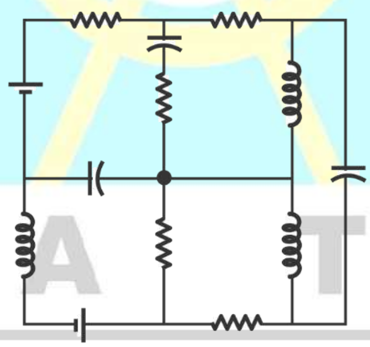
Key Point

(i) If the determinant $|A| = 0,$ then A is a singular matrix

(ii) If the determinant $|A| \neq 0,$ then A is a non singular matrix

Question 12 ***[Network Theory : Basic of Network]***

The number of junctions in the circuit is



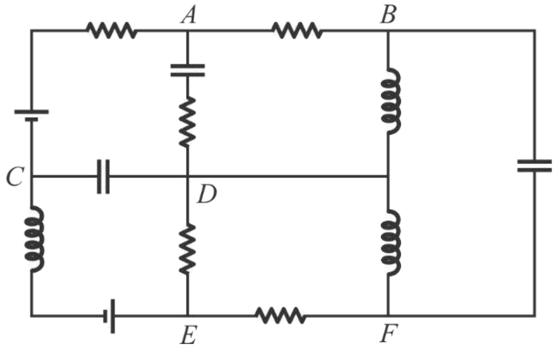
(A) 6 (B) 7

(C) 8 (D) 9

Ans. **(A)**

Sol. **Given :**

Given circuit is shown below.

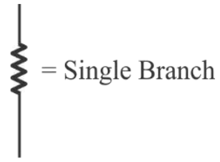


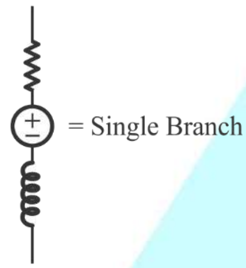
Concept :

- In an electrical network the junction of branches at a common point is called Node.
  A junction is a point where at three circuit path meet.
- Branch : A branch is a single element (or) component in a circuit.

If several elements in the circuit carrying same circuit they can also reffered as branch.

Example :

1. 

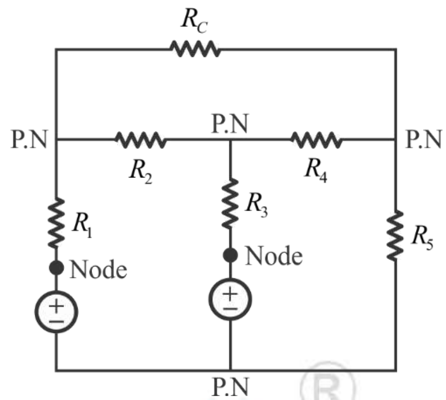
2. 

3. 

Note : A node is a point in a network where two or more elements are joined.

Principle node : Junction of three or more elements.

Example :



(•) Branches = 6

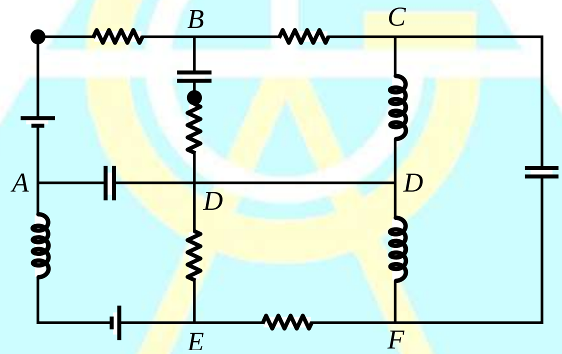
(•) Node = 2

(•) Principle Node = 4

Key Point :

A principal node is a junction where current can combine or divide.

Given : Circuit shown in below figure.



Number of branches $(b) = 10$

Number of Node $(N) = 3$

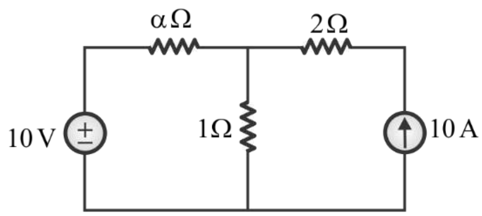
Number of principal node (or) junction = 6

A node is a point where two or more circuit elements are connected to it. If only two circuit elements are connected to node, then it is said to be simple node.

It three or more circuit elements are connected to a node then it is said to be principal node.

Question 13 ***[Network Theory : Basic of Network]***

All the elements in the circuits are ideal. The power delivered by the 10 V source in watts is

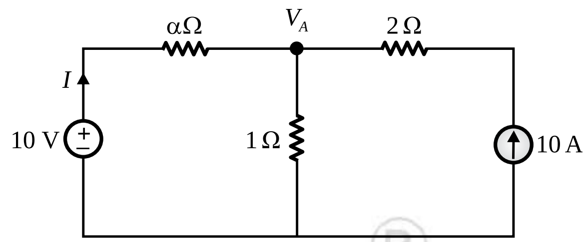


(A) 0 (B) 50

(C) 100 (D) Dependent on the value of α

Ans. **(A)**

Sol. Given circuit is shown below



Using nodal analysis :

Applying KCL at node V_A

$$\frac{V_A - 10}{\alpha} + \frac{V_A}{1} = 10$$

$$V_A - 10 + \alpha V_A = 10\alpha$$

$$(1+\alpha)V_A = (10\alpha + 10)$$

$$(1+\alpha)V_A = (\alpha + 1)10$$

$$V_A = 10\,\text{V}$$

Now we can say that there is no rate of resistance $\alpha\Omega$ therefore we can neglect this resistance.

$$I = \frac{10 - V_A}{\alpha} = 0\,\text{Amp}$$

The power delivered by the 10V source

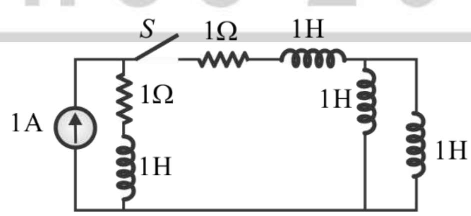
$$P_{10V(del)} = 10(I)$$

$$P_{10V(del)} = 10(0) = 0\,\text{Watt}$$

Hence, the correct option is (A).

Question 14 ***[Network Theory : Transient]***

The circuit shown in the figure with the switch S open, is in steady state. After the switch S is closed, the time constant of the circuit in seconds is

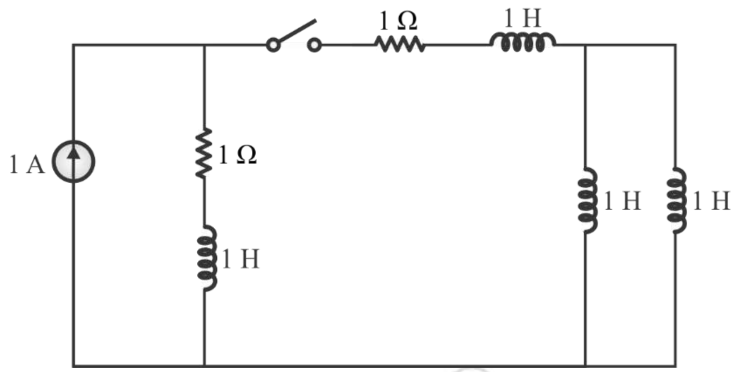


(A) 1.25 (B) 0

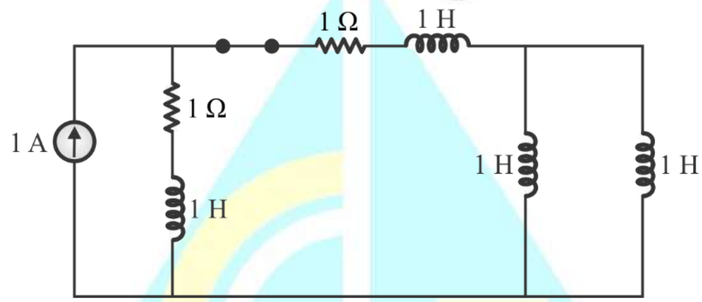
(C) 1 (D) 1.5

Ans. **(A)**

Sol. Given circuit is shown below.



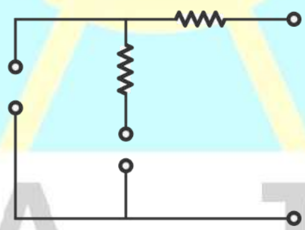
For $t \geq 0$ (transient) :



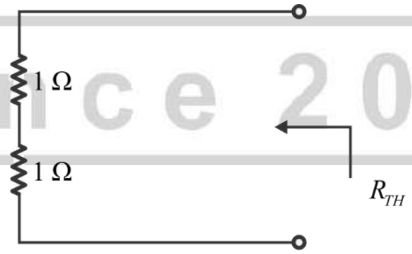
For R-L Network :

Time constant $\tau = \frac{L_{eq}}{R_{eq}} = \frac{L_{eq}}{R_{TH}}$

Where R_{Th} = Thevenin resistance across the inductor where all independent sources are deactivated.

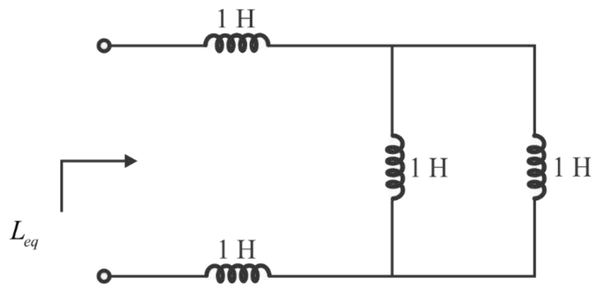


Redraw (or) modify the network



$R_{Th} = 1 + 1 = 2\Omega$

- For L_{eq} :



$$L_{eq} = 1 + 1 + \frac{1}{2} = \frac{5}{2}\text{ H}$$

Time constant $\frac{L_{eq}}{R_{Th}} = \frac{5}{2 \times 2} = \frac{5}{4} = 1.25$

Hence, the correct option is (A).

Question 15 ***[Signals & Systems : Convolution]***

Suppose signal $y(t)$ is obtained by the time-reversal of signal $x(t)$. i.e., $y(t) = x(-t), -\infty < t < \infty$. Which one of the following options is always true for the convolution of $x(t)$ and $y(t)$?

(A) It is an even signal

(B) It is an odd signal

(C) It is a causal signal

(D) It is an anti - causal signal

Ans. (A)

Sol. Given :

If $y(t) = x(-t)$, then

$$y(t) * x(t) = ?$$

Let :

$$z(t) = y(t) * x(t) \qquad \ldots (1)$$

(i) Time scaling property :

$$x(at) * y(at) = \frac{1}{|a|} z(at)$$

Put, $a = -1$

$$x(-t) * y(-t) = z(-t)$$

(ii) Commutative property :

$$x(t) * y(t) = y(t) * x(t)$$

From equation (1)

$$z(t) = y(t) * x(t) = x(-t) * x(t) \qquad \ldots . (2)$$

Put, $t = -t$

$$z(-t) = y(-t) * x(-t)$$

$$z(-t) = x(t) * x(-t)$$

From commutative property :

$$z(-t) = x(-t) * x(t) \qquad \text{.... (3)}$$

From equation (2) and (3)

$$z(t) = z(-t)$$

Hence the correct option is (A).

Question 16 ***[Signals & Systems : Laplace Transform]***

If $u(t)$ is the unit step function, then the region of convergence (ROC) of the Laplace transform of the signal.

$x(t) = e^{t^2}[u(t-1) - u(t-10)]$ is

(A) $-\infty < \text{Re}(s) < \infty$ (B) $\text{Re}(s) \geq 10$

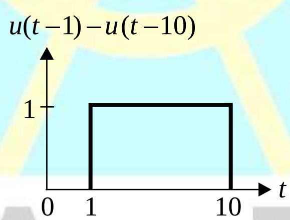
(C) $\text{Re}(s) \leq 1$ (D) $1 \leq \text{Re}(s) \leq 10$

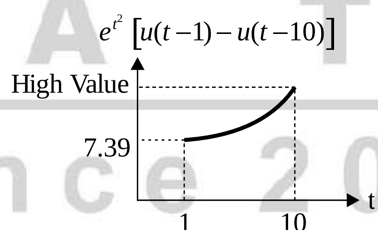
Ans. (A)

Sol. Given :

$$x(t) = e^{t^2}\left[u(t-1) - u(t-10)\right]$$

$$x(t) = \left(e^2\right)^t\left[u(t-1) - u(t-10)\right]$$





This is finite duration signal.

$\Rightarrow$ ROC of transform of a bounded finite duration signal is entire s-plane.

Hence the correct option is (A).

Question 17 ***[Machine : Induction Machine]***

A three phase, 50 Hz, 6 pole induction motor runs at 960 rpm. The stator copper loss, core loss and the rotational loss of the motor can be neglected. The percentage efficiency of the motor is

(A) 92 (B) 94

(C) 96 (D) 98

Ans. **(C)**

Sol. **Given :**

(i) $f = 50\ Hz$

(ii) $P = 6$

(iii) $N_r = 960$ rpm

(iv) The stator copper loss and the rotational loss of the motor can be neglected.

$$\text{Slip } (s) = \frac{N_s - N_r}{N_s}$$

$$N_s = \frac{120\, f}{P} = \frac{120 \times 50}{6} = 1000 \text{ rpm}$$

$$\text{Slip } (s) = \frac{1000 - 960}{1000} = 0.04$$

$$\text{Efficiency } (\eta) = \frac{P_{out}}{P_{in}} = (1 - s)$$

$$\eta = (1 - 0.04) = 0.96$$

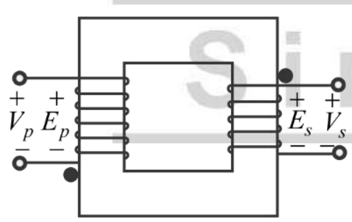
$$\%(\eta) = 96\%$$

Hence the correct option is (C).

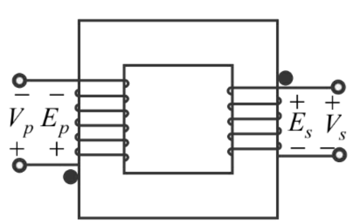
Question 18 **[Machine : Transformer]**

Which one of the following options represents possible voltage polarities in a single phase two winding transformer? Here. V_p is the applied primary voltage. E_p is the induced primary voltage. V_s is the open circuit secondary voltage, and E_s is the induced secondary voltage.

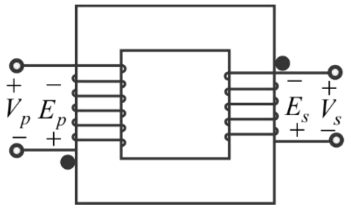
(A)



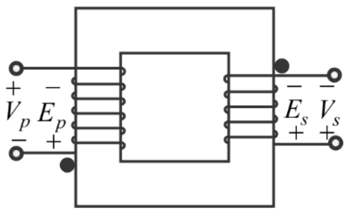
(B)



(C)



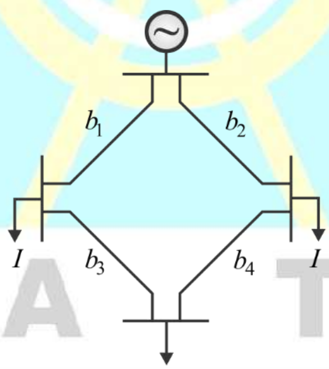
(D)



Ans. **MTA**

Question 19 ***[Power System]***

The figure show the single line diagram of a 4 - bus power network. Branches b_1, b_2, b_3 and b_4 have impedance $4z, z, 2z$ and $4z$ per - unit (pu), respectively, where $z = r + jx,$ with $r > 0$ and $x > 0$. The current drawn from each load bus (marked as arrows) is equal to I pu, where $I \neq 0$. If the network to operate with minimum loss, the branch that should be opened is



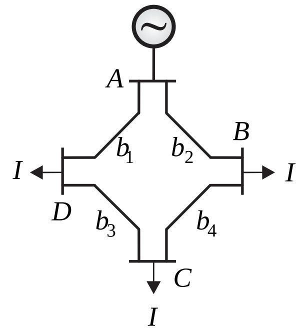
(A) b_1 (B) b_2

(C) b_3 (D) b_4

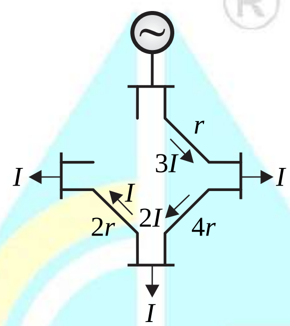
Ans. **(C)**

Sol. **Given :**

Branches b_1, b_2, b_3 and b_4 have impedance $4z, z, 2z,$ and $4z$ per unit respectively, where $z = r + jx$ with $r > 0$ and $x > 0$.



(i) If b_1 branch is removed :

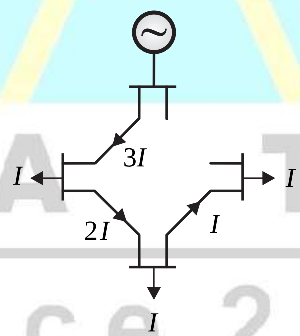


$$\text{Total power loss } (P_{L_1}) = (3I)^2 r + (2I)^2 4r + (I)^2 2r$$

$$= (9+16+2) I^2 r$$

$$P_{L_1} = 27 I^2 r$$

(ii) If b_2 branch is removed :

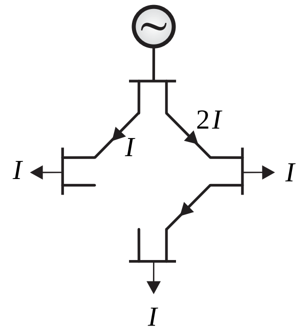


$$\text{Total power loss } (P_{L_2}) = (3I)^2 (4r) + (2I)^2 (2r) + (I)^2$$

$$= I^2 r (36+8+4)$$

$$P_{L_2} = 48 I^2 r$$

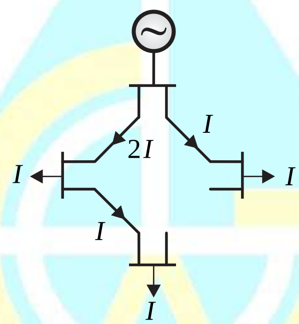
(iii) If b_3 branch is removed :



Total power loss $\left(P_{L_3}\right)=(I)^2\,4r+(2I)^2\,r+(I)^2\,4r$

$$P_{L_3}=12I^2r$$

(iv) If b_4 branch is removed :



Total power loss $\left(P_{L_4}\right)=(2I)^2\,4r+(I)^2(2r)+(I)^2$

$$=I^2r(16+2+1)$$

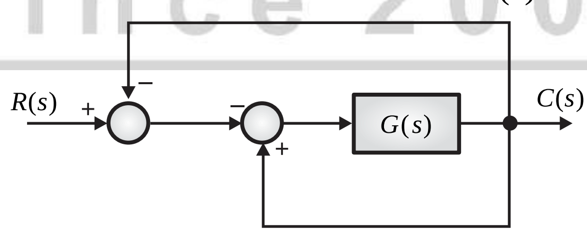
$$P_{L_4}=19I^2r$$

By opening branch b_3 power loss is minimum.

Hence the correct option is (C).

Question 20 **[Control System : Block Diagram]**

For the block - diagram shown in figure, the transfer function $\dfrac{c(s)}{R(s)}$ is



(A) $\dfrac{G(s)}{1+2G(s)}$

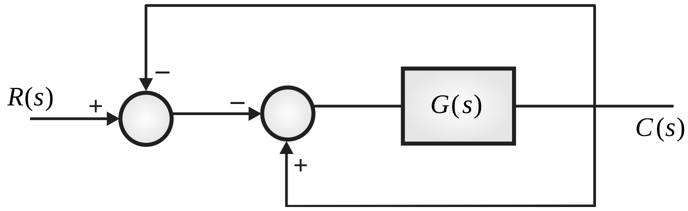
(B) $-\dfrac{G(s)}{1+2G(s)}$

(C) $\dfrac{G(s)}{1-2G(s)}$

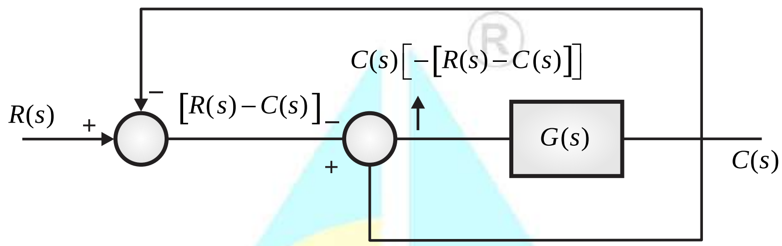
(D) $-\dfrac{G(s)}{1-2G(s)}$

Ans. **(D)**

Sol. Given block diagram is shown below



From block diagram



$$C(s) = G(s)[C(s) - R(s) + C(s)]$$

$$C(s) = G(s)[2C(s) - R(s)]$$

$$C(s) = 2G(s)C(s) - G(s)R(s)$$

$$C(s)[1 - 2G(s)] = -G(s)R(s)$$

$$\frac{C(s)}{R(s)} = \frac{-G(s)}{1 - 2G(s)}$$

Hence, the correct option is (D).

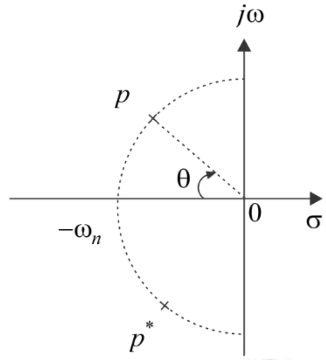
Question 21 ***[Control System : Second Order System]***

Consider the standard second - order system of the form $\frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$ with the poles p and p^* having negative real parts. The pole locations are also shown in the figure. Now consider two such second - order systems as defined below :

System 1 : $\omega_n = 3 \text{rad/sec}$ and $\theta = 60^0$

System 2 : $\omega_n = 1 \text{rad/sec}$ and $\theta = 70^0$

Which one of the following statements is correct ?



(A) Settling time of system 1 is more than that of system 2.

(B) Settling time of system 2 is more than that of system 1.

(C) Settling times of both the systems are the same.

(D) Settling time cannot be computed from the given information.

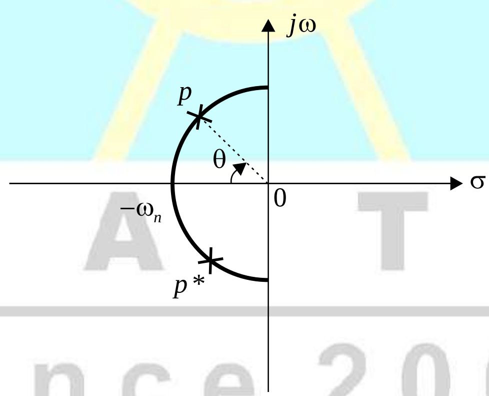
Ans. (B)

Sol. **Given :** Consider the standard second order system of the form $\dfrac{\omega n^2}{s^2 + 2\xi\omega_n S + \omega_n^2}$ with the pole P and P^* having negative real parts.

System 1 : $\omega_n = 3\text{rad/sec}$ and $\theta = 60^0$

System 2 : $\omega_n = 1\text{rad/sec}$ and $\theta = 70^0$

The pole location are also shown in the below figure.



Setting Time : Time required by the response to reach and steady within specified range of % to 5% of its final value.

For 2% tolerance : $t_s = 4\tau = \dfrac{4}{\xi\omega_n}$ (Approx)

For 5% tolerance : $t_s = 3\tau = \dfrac{3}{\xi\omega_n}$ (Approx)

Where $\tau =$ Time constant

$$\tau = \frac{1}{\xi \omega_n}$$

Key Point :

1. For all positive value of ξ system are static.
2. Pole away from origin are called insignificant pole and does not affect the stability of the system.
3. $\theta = \cos^{-1}(\xi)$

According to question :

System 1 : $\omega_n = 3 \text{rad/sec}$

$$\omega_n = 3 \text{rad/sec}$$

$$\theta = \cos^{-1}(\xi)$$

$$\xi = \cos\theta = \cos(60^0) = 0.5$$

$$t_{s(1)} = \frac{4}{0.5 \times 3} = 2.67 \sec$$

System 2 :

$$\omega_n = 1 \text{rad/sec}$$

$$\xi = \cos\theta = \cos(70^0)$$

$$\xi = 0.34$$

$$t_{s_{(2)}} = \frac{4}{0.34 \times 1} = 11.76 \sec$$

Hence, the correct option is (B)

Question 22 ***[Control System]***

Consider the cascaded system as shown in the figure. Neglecting the faster component of the transient response, which one of the following options is a first - order pole - only approximation such that the steady – state values of the unit step response of the original and the approximated systems are same ?

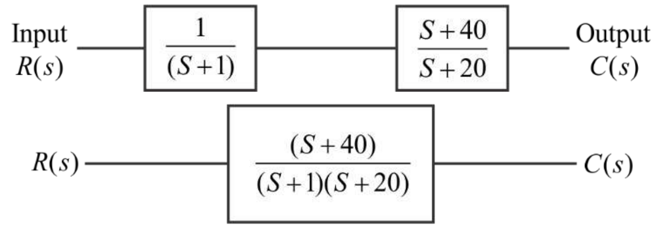
(A) $\frac{1}{s+1}$ (B) $\frac{2}{s+1}$

(C) $\frac{1}{s+20}$ (D) $\frac{2}{s+20}$

Ans. (B)

Sol. Given :

The cascaded system as shown in the below figure.



Before approximation :

$$T(s)=\frac{C}{R}=\frac{(S+40)}{(S+1)(S+20)}$$

$$\text{DC gain } (S=0)=\frac{40}{20}=2$$

After approximation :

$$T(s)=\frac{40\left(\frac{S}{40}+1\right)}{(S+1)20\left(\frac{S}{20}+1\right)}$$

DC gain $(S=0)=2$

$$T(s)=\frac{2}{(S+1)}$$

Hence, the correct option is (B).

Key Point

For a periodic

- The value of DC gain should be some after approximation.
- For approximation of transfer function. We can neglect the insignificant pole.
- For existence of the dominant pole concept the ratio of the insignificant pole to the significant should be greater than equal to 4
- Significant should is also referred as the dominant pole and exist nearer to the origin on the left hand side of the $j\omega$ - Axis for a stable system

Question 23 ***[Measurement : (CT/PT)]***

The table lists two instrument transformers and their features :

Instrument transformers		Features	
		(P)	Primary is connected in parallel to the grid
(X)	Current transformer (CT)	(Q)	Open circuited secondary is not desirable
(Y)	Potential transformer (PT)	(R)	Primary current is the line current
		(S)	Secondary burden affects the primary current

(A) X matches with P and Q; Y matches with R and S.

(B) X matches with P and R; Y matches with Q and S.

(C) X matches with Q and R; Y matches with P and S.

(D) X matches with Q and S; Y matches with P and R.

Ans. (C)

Question 24 ***[Digital Electronics : K-Map]***

Simplified form of the Boolean function

$$F(P,Q,R,S)=\bar{P}\bar{Q}+\bar{P}QS+P\bar{Q}\bar{R}\bar{S}+P\bar{Q}R\bar{S} \text{ is}$$

(A) $\bar{P}S+\bar{Q}\bar{S}$ (B) $\bar{P}\bar{Q}+\bar{Q}\bar{S}$

(C) $\bar{P}Q+R\bar{S}$ (D) $P\bar{S}+Q\bar{R}$

Ans. (A)

Sol. Given function F is in SOP form :

$$F\left(P,Q,R,S\right)=\bar{P}\bar{Q}+\bar{P}QS+P\bar{Q}\bar{R}\bar{S}+P\bar{Q}R\bar{S}$$

Method 1

$$F\left(P,Q,R,S\right)=\bar{P}\bar{Q}+\bar{P}QS+P\bar{Q}\bar{R}\bar{S}+P\bar{Q}R\bar{S}$$

$$=\bar{P}\bar{Q}+\bar{P}QS+P\bar{Q}\bar{S}\left(\bar{R}+R\right)$$

OR Law :

(1) $1+A=1$

(2) $A+\bar{A}=1$

(3) $A+A=A$

(4) $A+0=A$

$$F\left(P,Q,R,S\right)=\bar{P}\bar{Q}+\bar{P}QS+P\bar{Q}\bar{S}$$

$$=\bar{P}\bar{Q}\left(S+\bar{S}\right)+\bar{P}QS+P\bar{Q}\bar{S}$$

$$=\bar{P}\bar{Q}S+\bar{P}\bar{Q}\bar{S}+\bar{P}QS+P\bar{Q}\bar{S}$$

$$=\bar{P}S\left(Q+\bar{Q}\right)+\bar{Q}\bar{S}\left(P+\bar{P}\right)$$

$$=\bar{P}S+\bar{Q}\bar{S}$$

Hence the correct option is (A).

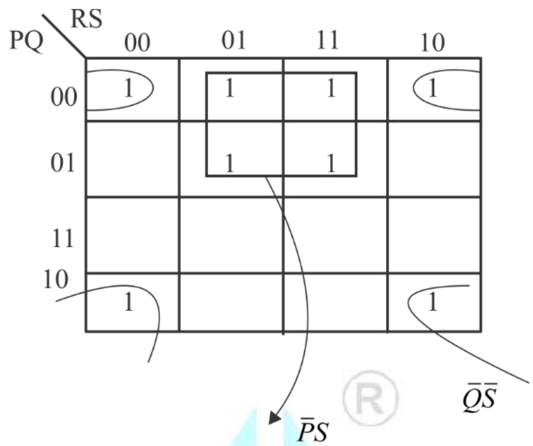
Method 2

$$F\left(P,Q,R,S\right)=\bar{P}\bar{Q}+\bar{P}QS+P\bar{Q}\bar{R}\bar{S}+P\bar{Q}R\bar{S}$$

$$=\bar{P}\bar{Q}\left(R+\bar{R}\right)\left(S+\bar{S}\right)+\bar{P}QS\left(R+\bar{R}\right)+P\bar{Q}\bar{R}\bar{S}+P\bar{Q}R\bar{S}$$

$$F\left(P,Q,R,S\right)=\bar{P}\bar{Q}\bar{R}\bar{S}+\bar{P}\bar{Q}\bar{R}S+\bar{P}\bar{Q}R\bar{S}+\bar{P}\bar{Q}RS+\bar{P}QSR+\bar{P}QS\bar{R}+P\bar{Q}\bar{R}\bar{S}+P\bar{Q}R\bar{S}$$

k-map of function f in SOP form is :

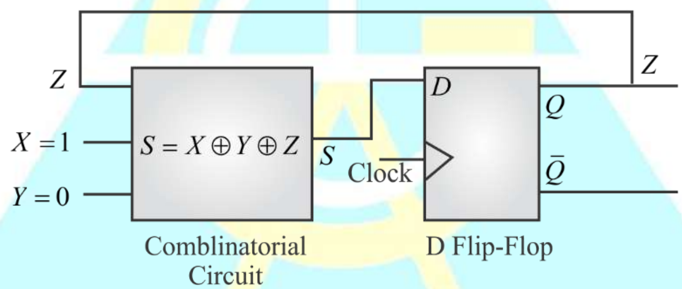


$$F(P,Q,R,S)=\bar{P}S+\bar{Q}\bar{S}$$

Hence the correct option is (A).

Question 25 ***[Digital Electronics]***

In the circuit, the present value of Z is 1. Neglecting the delay in the combinatorial circuit, the values of S and Z respectively, after the application of the clock will be

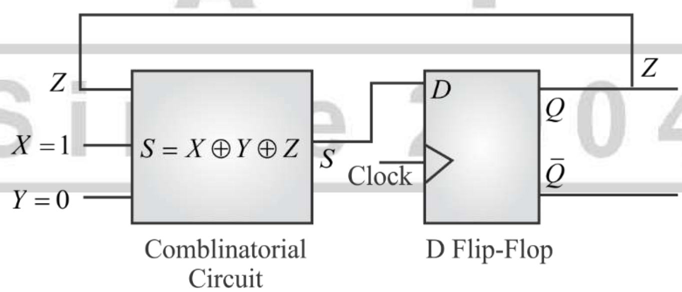


(A) $S=0, Z=0$ (B) $S=0, Z=1$

(C) $S=1, Z=0$ (D) $S=1, Z=1$

Ans. **(C)**

Sol. **Given circuit is shown below.**



- Initially $z=1$

  $S=x\oplus y\oplus z$

  $S=(x\oplus y)\oplus z$

  $S=(x\bar{y}+\bar{x}y)\oplus z$

  $[\because x=1, y=0, z=1]$

$S = (1 \oplus z) = (1\bar{z} + 0)$

$S = \bar{z} = 0$

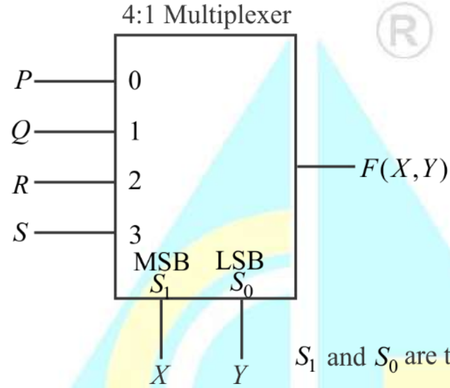
- One clock pulse :

  $Q = D = S = 0$

Hence the Correct option is (C).

Question 26 ***[Digital Electronics : Combination Circuit]***

To obtain the Boolean function $F(X,Y) = X\bar{Y} + \bar{X}$, the inputs PQRS in the figure should be



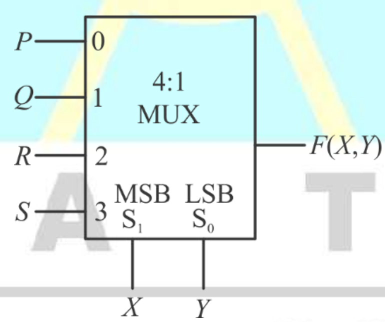
S_1 and S_0 are the select lines

(A) 1010 (B) 1110

(C) 0110 (D) 0001

Ans. (B)

Sol. Given : $F(X,Y) = X\bar{Y} + \bar{X}$



Truth table is shown below :

X	Y	Selected Input	F
0	0	I_0	P
0	1	I_1	Q
1	0	I_2	R
1	1	I_3	S

$F = X\overline{Y} + \overline{X}$ ...(i)

The output of $4X1$ Mux is given by,

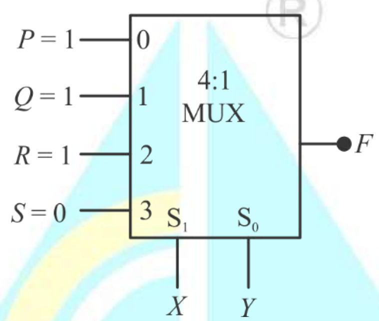
$$F = \overline{S}_1\overline{S}_0 I_0 + \overline{S}_1 S_0 I_1 + S_1\overline{S}_0 I_2 + S_1 S_0 I_3$$

$$F = \overline{X}\overline{Y} I_0 + \overline{X} Y I_1 + X\overline{Y} I_2 + XY I_3$$

$$F = \overline{X}\overline{Y}P + \overline{X}YQ + X\overline{Y}R + XYS \quad \ldots\text{(ii)}$$

Comparing equation (i) and (ii)

$P = 1,\ Q = 1,\ R = 1,\ S = 0$



Hence the correct option is (B).

Question 27 ***[Power Electronics]***

If the following switching devices have similar power ratings, which one of them is the fastest ?

(A) SCR (B) GTO

(C) IGBT (D) Power MOSFET

Ans. (D)

Sol. Power MOSFET :

(1) It has very fast switching speed

(2) A MOSFET has a shorter turn – OFF time than an IGBT

(3) Compared to IGBT. MOSFET has a larger forward voltage drop.

Hence the correct option is (D)

Question 28 ***[Power Electronics : AC Voltage Controller]***

A single - phase triac based AC voltage controller feeds a series RL load. The input AC supply is 230 V, 50 Hz. The values of R and L are $10\,\Omega$ and 18.37 mH, respectively. The minimum triggering angle of the triac to obtain controllable output voltage is

(A) 15^0 (B) 30^0

(C) 45^0 (D) 60^0

Ans. (B)

Sol. **Given :**

(i) $R = 10\,\Omega$

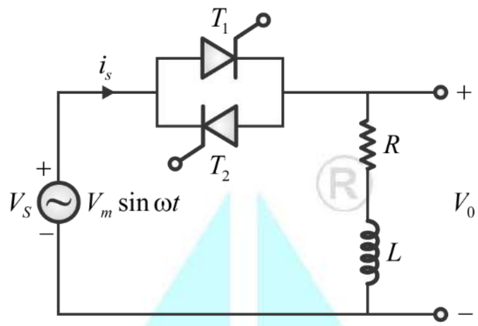
(ii) $L = 18.37\,\text{mH}$

(iii) $f = 50\,\text{Hz}$

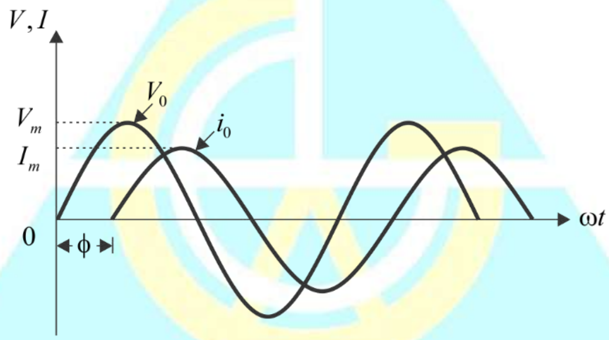
(iv) AC supply = 230 V

RL – Load

Case 1 : $(\alpha > \phi)$, V_0 is controlled .



In steady state current will lag voltage by certain angle ϕ as shown in figure below



Where $\phi = \tan^{-1}\left(\frac{\omega L}{R}\right)$

$$\phi = \tan^{-1}\left(\frac{2\pi \times 50 \times 18.37 \times 10^{-3}}{10}\right)$$

$\phi = 29.989^0$

Hence the correct option is (B)

Question 29 [MSQ] ***[Mathematics : Probability]***

Let X be a discrete random variable that is uniformly distributed over the set $\{-10, -9, ..., 0, ..., 9, 10\}$. Which of the following random variables is/are uniformly distributed ? **(1 Mark)**

(A) X^2 (B) X^3

(C) $(X-5)^2$ (D) $(X+10)^2$

Ans. **(B, D)**

Sol. **Given :** X is discrete random variable that is uniformly distributed over the set

$$\{-10, -9, ..., 0, ..., 9, 10\}$$

Probability of getting either -10 or -9 or probability of getting either 0, or 9 or 10, all are same i.e. $\frac{1}{(21)}$.

From option (A) :

$X^2 \to \{100, 81, ..., 0, ...81, 100\}$

Here, probability of getting every number are not same because all the values are not different, therefore X^2 is not uniformly distributed.

From option (B) :

$X^3 \to \{-1000, -729, ..., 0, ...729, 1000\}$

Similarly, probability of getting every numbers are same because all the values are different. Therefore, X^3 is uniformly distributed.

From option (C) :

$$(X-5)^2 \to \{(-15)^2, (-14)^2, ...(-5)^2, ...(4)^2, (5)^2\}$$

$$\to \{225, 196, ...25, ...16, 25\}$$

Here, probability of getting every numbers are not same because all values are not different. Therefore, $(X-5)^2$ is not uniformly distributed.

From option (D) :

$$(X+10)^2 \to \{0, (1)^2, ...(10)^2, ...(19)^2, (20)^2\}$$

Here, probability of getting every numbers are same because all values are different.

Hence, the correct options are (B) and (D).

Question 30 [MSQ] ***[Mathematics : Complex Variable]***

Which of the following complex functions is/are analytic on the complex plane ? **(1 Mark)**

(A) $f(z) = j\,\mathrm{Re}(z)$ (B) $f(z) = \mathrm{Im}(z)$

(C) $f(z) = e^{|z|}$ (D) $f(z) = z^2 - z$

Ans. (D)

Sol. Given :

$$z = x + jy$$

For the function to be analytic. It must satisfy Cauchy – Riemann equations.

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}, \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$$

Checking from options :

From option (A) :

$$f(z) = j\,\mathrm{Re}(z)$$

$$f(z)=jx$$

Since, there is only imaginary part. Therefore Cauchy – Riemann equation does not satisfied.

From option (B) :

$$f(z)=\mathrm{Im}(z)$$

$$f(z)=jy$$

Since, there is only imaginary part. Therefore Cauchy – Riemann equation does not satisfied.

From option (C) :

$$f(z)=e^{|z|}$$

$$z=x+jy$$

$$|z|=\sqrt{x^2+y^2}$$

$$f(z)=e^{\sqrt{x^2+y^2}}$$

Since, there is only real part. Therefore, Cauchy – Riemann equation does not satisfied.

From option (D) :

$$f(z)=\left(z^2-z\right)$$

$$=\left(x+jy\right)^2-\left(x+jy\right)$$

$$=x^2-y^2+2jxy-x-jy$$

$$f(z)=\left(x^2-y^2-x\right)+j\left(2xy-y\right)$$

$$f(z)=u+jv$$

Here, $u=\left(x^2-y^2-x\right)$

$$v=\left(2xy-y\right)$$

$$\frac{\partial u}{\partial x}=\left(2x-1\right),\quad \frac{\partial v}{\partial y}=2x-1$$

$$\frac{\partial u}{\partial x}=\frac{\partial v}{\partial y}=(2x-1)$$

$$\frac{\partial u}{\partial y}=-2y \qquad \frac{-\partial v}{\partial x}=-2y$$

$$\frac{\partial u}{\partial y}=\frac{-\partial v}{\partial x}=\left(-2y\right)$$

Hence, Cauchy – Riemann equation is satisfied.

Hence the correct option is (D).

Key Point

(1) If $f(z)$ and $g(z)$ are two analytic functions.
Then,
(i) $f(z) \pm g(z)$ will be analytic function.
(ii) $\dfrac{f(z)}{g(z)}$ will be analytic function if $g(z) \neq 0$.
(iii) $f(z) \times g(z)$ will be an analytic function.
(2) Polynomial and Exponential functions are entire functions.

Question 31 ***[Mathematics : Limit Series]***

Considered the complex function $f(z) = \cos z + e^{z^2}$. The coefficient of z^5 in the Taylor series expansion of $f(z)$ about the origin is _______ (rounded off to 1 decimal place). **(1 Mark)**

Ans. **0.0 to 0.0**

Sol. **Given :** $f(z) = \cos z + e^{z^2}$

Series expansion :

$$e^z = 1 + \frac{z}{1!} + \frac{z^2}{2!} + \frac{z^3}{3!} + ...\infty$$

$$\sin z = z - \frac{z^3}{3!} + \frac{z^5}{5!} - \frac{z^7}{7!} + ...$$

$$\cos z = 1 - \frac{z^2}{2!} + \frac{z^4}{4!} - \frac{z^6}{6!} + ...$$

$$e^{z^2} = 1 + \frac{z^2}{1!} + \frac{z^4}{2!} + \frac{z^6}{3!} + ...$$

$$f(z) = \left(1 - \frac{z^2}{2!} + \frac{z^4}{4!} - \frac{z^6}{6!} + ...\right) + \left(1 + \frac{z^2}{1!} + \frac{z^4}{2!} + \frac{z^6}{3!} + ...\right)$$

Since there is no term of the power of z^5.

Hence, the coefficient of z^5 will be zero.

Question 32 ***[Mathematics : Linear Algebra]***

The sum of the eigenvalues of the matrix $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}^2$ is _________(Rounded off to the nearest integer)

(1 Mark)

Ans. **29 to 29**

Sol. **Given :**

$$A=\begin{bmatrix}1 & 2\\3 & 4\end{bmatrix}^2$$

$$A=\begin{bmatrix}1 & 2\\3 & 4\end{bmatrix}\begin{bmatrix}1 & 2\\3 & 4\end{bmatrix}$$

$$A=\begin{bmatrix}(1\times1)+(3\times2) & (1\times2)+(4\times2)\\(1\times3)+(3\times4) & (3\times2)+(4\times4)\end{bmatrix}$$

$$A=\begin{bmatrix}7 & 10\\15 & 22\end{bmatrix}$$

Method 1

The characteristic equation is given by

$$|A-\lambda I|=0$$

$$\begin{vmatrix}(7-\lambda) & 10\\15 & (22-\lambda)\end{vmatrix}=0$$

$$(7-\lambda)(22-\lambda)-150=0$$

$$154-7\lambda-22\lambda+\lambda^2-150=0$$

$$\lambda^2-29\lambda+4=0$$

$$\lambda=\frac{29\pm\sqrt{841-(4\times4)}}{2}$$

$$\lambda=\frac{29\pm\sqrt{841-16}}{2}$$

$$\lambda=\frac{29+\sqrt{825}}{2}=\frac{29+28.723}{2}$$

$$\lambda_1=28.8615$$

$$\lambda_2=0.1385$$

The sum of Eigen values $=(28.8615+0.1385)=29$

Method 2

Given :

$$A=\begin{bmatrix}1 & 2\\3 & 4\end{bmatrix}^2$$

$$A=\begin{bmatrix}7 & 10\\15 & 22\end{bmatrix}$$

Now according to the first properties.

$$(\lambda_1+\lambda_2)= \text{Trace of matrix } A$$

$$(\lambda_1+\lambda_2)=(7+22)=29$$

$$(\lambda_1+\lambda_2)=29$$

Key Point

Properties of Eigen values or characteristic roots

(i) The sum of Eigen values of a matrix is equal to the trace of the matrix, where the sum of the elements of Principal diagonal of a matrix is called the trace of matrix.

$$\sum_i \lambda_i = \lambda_1+\lambda_2+\lambda_3 \ldots\ldots = \text{Trace of matrix}$$

(ii) The product of Eigen values of matrix A is equal to the determinant of matrix A.

Question 33 ***[Signals & Systems : Fourier Transform]***

Let $X(\omega)$ be the Fourier transform of the signal

$$x(t)=\mathrm{e}^{-t^4}\cos t,\ -\infty<t<\infty$$

The value of the derivative of $X(\omega)$ at $\omega=0$ is __________ (Rounded off to 1 decimal place).

Ans. 0.0 to 0.0

Sol. Given :

$$x(t)=e^{-t^4}\cos(t)\quad -\infty<t<\infty$$

$$\left.\frac{dx(\omega)}{d\omega}\right|_{\omega=0}=?$$

Fourier Transform :

$$x(\omega)=\int_{-\infty}^{\infty} x(t)e^{-j\omega t}\,dt$$

$$\frac{dx(\omega)}{d\omega}=\int_{-\infty}^{\infty} x(t)(-jt)e^{-j\omega t}\,dt$$

$$\left.\frac{dx(\omega)}{d\omega}\right|_{\omega=0}=\int_{-\infty}^{\infty} x(t)[-jt]\,dt$$

$$\left.\frac{dx(\omega)}{d\omega}\right|_{\omega=0}=\int_{-\infty}^{\infty} (-jt)e^{-t^4}\cos t\,dt$$

(i) $e^{-t^4}\cos t=\text{Even}\times\text{Even}=\text{Even}$

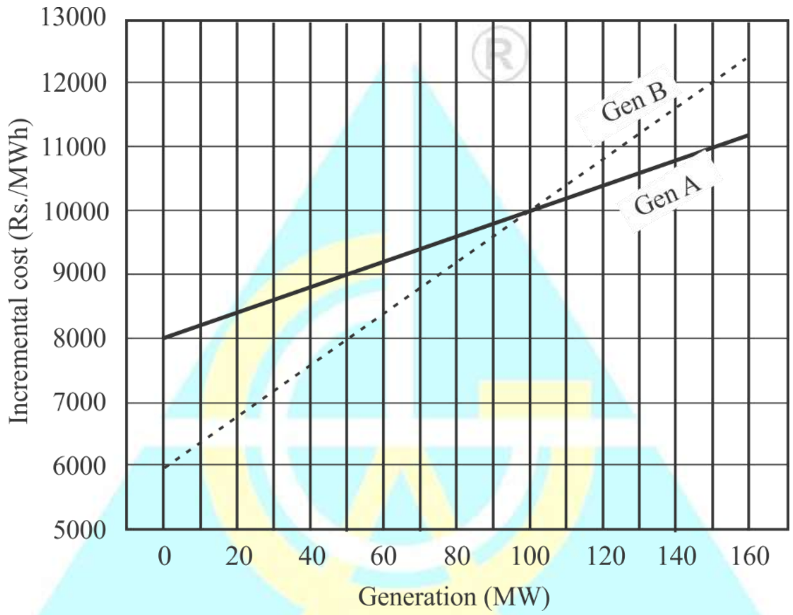
(ii) $(-jt)\left[e^{-t^4}\cos t\right]=\text{odd}\times\text{Even}=\text{Odd}$

$$\left.\frac{dx(\omega)}{d\omega}\right|_{\omega=0} = \int_{-\infty}^{\infty}(\text{odd signal})\,dt = 0$$

Hence the correct answer is (0).

Question 34

The incremental cost curves of two generators (Gen A and Gen B) in a plant supplying a common load are shown in the figure. If the incremental cost of supplying the common load is Rs. 7400 per MWh, then the common load in MW is __________ (rounded off to the nearest integer).



Ans. 35 to 35

Sol. **Given :** $\lambda = 7400\,\text{Rs / MWhr}$

Gen (A) :

$$(IC)_A = \frac{(11000-10000)}{(150-100)}P_{GA} + 8000$$

$$(IC)_A = \frac{1000}{50}P_{GA} + 8000$$

$$(IC)_A = 20P_{GA} + 8000 \qquad \text{....(i)}$$

Gen (B) :

$$(IC)_B = \frac{(12000-10000)}{50}P_{GB} + 6000$$

$$(IC)_B = 40P_{GB} + 6000 \qquad \text{....(ii)}$$

Given :

$$\lambda = (IC)_A = (IC)_B = 7400 \qquad \text{....(iii)}$$

$$40\,P_{GA} + 6000 = 7400$$

$$40 P_{GA} = 1400$$

$$P_{GA} = 35\,\text{MW}$$

From equation (iii)

$$20P_{GA} + 8000 = 40(35) + 6000$$

$$20P_{GA} + 8000 = 7400$$

$$20P_{GA} = -600$$

$$20P_{GA} = -30\,\text{MW}$$

$$P_{GA} < 0$$

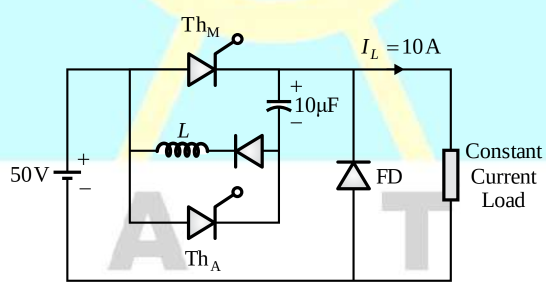
Common Load $= P_{GA} + P_{GB} = 0 + 35$

= 35 MW

Hence the correct Answer is (35).

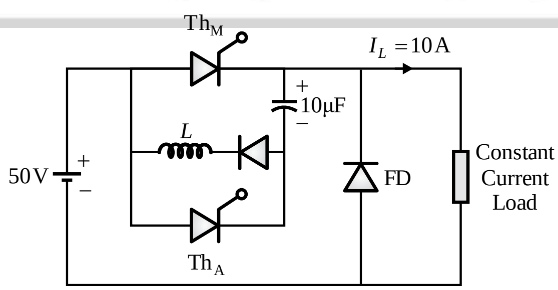
Question 35 **[Power Electronics : Chopper]**

A forced commutated thyristorized step - down chopper is shown in the figure. Neglect the ON - state drop across the power devices. Assume that the capacitor is initially charged to 50 V with the polarity shown in the figure. The load current (I_L) can be assumed to be constant at 10 A. Initially, Th_M is ON and Th_A is OFF. The turn – off time available to Th_M in microseconds, when Th_A is triggered, is _______(rounded off to the nearest integer)



Ans. 50 to 50

Sol. Given Circuit is Shown below



$$t_{cm} = \frac{CV_S}{I_0} = \frac{10\times10^{-6}\times50}{10}$$

$$t_{cm} = 50\ \mu\text{sec}$$

Hence the correct Answer is 50.

Q.36 to Q.65 Carry TWO Marks Each

Question 36 **[Mathematics : Vector Calculus]**

Consider a vector $\bar{u} = 2\hat{x} + \hat{y} + 2\hat{z}$, where $\hat{x}, \hat{y}, \hat{z}$ represent unit vectors along the coordinate axes x, y, z respectively. The directional derivative of the function $f(x, y, z) = 2\ln(xy) + \ln(yz) + 3\ln(xz)$ at the point $(x, y, z) = (1,1,1)$ in the direction of $\bar{u}$ is : **(2 Marks)**

(A) 0

(B) $\frac{7}{5\sqrt{2}}$

(C) 7

(D) 21

Ans. (C)

Sol. Given : $f(x, y, z) = 2\ln(xy) + \ln(yz) + 3\ln(xz)$

Vector $\bar{u} = 2\hat{x} + \hat{y} + 2\hat{z}$

Gradient of a scalar function f is given by.

$$\nabla f = \hat{x}\frac{\partial f}{\partial x} + \hat{y}\frac{\partial f}{\partial y} + \hat{z}\frac{\partial f}{\partial z}$$

$$\nabla f = \hat{x}\frac{\partial}{\partial x}\left[2\ln(xy) + \ln(yz) + 3\ln(xz)\right] + \hat{y}\frac{\partial}{\partial y}\left[2\ln(xy) + \ln(yz) + 3\ln(xz)\right] +$$

$$\hat{z}\frac{\partial}{\partial z}\left[2\ln(xy) + \ln(yz) + 3\ln(xz)\right]$$

$$\nabla f = \hat{x}\left[\frac{2}{x} + \frac{3}{x}\right] + \hat{y}\left[\frac{2}{y} + \frac{1}{y}\right] + \hat{z}\left[\frac{1}{z} + \frac{3}{z}\right]$$

At point $(1,1,1)$

$$\nabla f\Big|_{(1,1,1)} = 5\hat{x} + 3\hat{y} + 4\hat{z}$$

Directional derivative in the direction of vector $\vec{u}$ is given by .

$$(\nabla f).\frac{\vec{u}}{|\vec{u}|} = \frac{(5\hat{x} + 3\hat{y} + 4\hat{z}).(2\hat{x} + \hat{y} + 2\hat{z})}{\sqrt{4+4+1}}$$

$$= \frac{10+3+8}{\sqrt{9}} = \frac{21}{3} = 7$$

Hence the correct option is (C).

Question 37 ***[Signals & Systems]***

The input $x(t)$ and the output $y(t)$ of a system are related as

$$y(t) = e^{-t}\int_{-\infty}^{t} e^{\tau} x(\tau)\, d\tau, \qquad -\infty < t < \infty.$$

The system is

(A) Nonlinear (B) Linear and time-invariant

(C) Linear but not time-invariant (D) Noncausal

Ans. (B)

Question 38 ***[Signals & Systems]***

Consider the discrete-time systems T_1 and T_2 defined as follows :

$$\{T_1 x\}[n] = x[0] + x[1] + \ldots + x[n]$$

$$\{T_2 x\}[n] = x[0] + \frac{1}{2}x[1] + \ldots + \frac{1}{2^n}x[n]$$

Which of the following statements is true ?

(A) T_1 and T_2 are BIBO stable

(B) T_1 and T_2 are not BIBO stable

(C) T_1 is BIBO stable but T_2 is not BIBO stable

(D) T_1 is not BIBO stable but T_2 is BIBO stable

Ans. (D)

Sol. **Given :**

$$\{T_1 x\}[n] = x[0] + x[1] + \ldots + x[n]$$

$$\{T_2 x\}[n] = x[0] + \frac{1}{2}x[1] + \ldots + \frac{1}{2^n}x[n]$$

For stability of discrete - Time System :

$$\sum_{k=-\infty}^{\infty} |h[k]| < \infty$$

$$\{T_1 x\}[n] = \sum_{k=0}^{n} x[k]$$

$$x[k] = u[k] = \text{ Bounded Input}$$

$$\{T_1 x\}[n] = \left|\sum_{k=0}^{n} u[k]\right| = \sum_{k=0}^{n} 1 = n$$

$\{T, x\}[n]|_{n=\infty} = \infty$ (System is BIBO stable)

$$\{T_2 x\}[n] = x[0] + \frac{1}{2}x[1] + \cdots + \frac{1}{2^n} \times [n]$$

$$=\sum_{k=0}^{n}\left(\frac{1}{2}\right)^{k} x[k]$$

$$x[k] = U[k]$$

$$\{T_2x\}[n] = \sum_{k=0}^{n}\left|\left(\frac{1}{2}\right)^{k} U(k)\right|$$

$$=\sum_{k=0}^{n}\left(\frac{1}{2}\right)^{k}$$

$$\{T_2x\}[n] = \frac{1}{\left(1-\frac{1}{2}\right)} = 2 \qquad \text{[Finite value]}$$

$\therefore T_2$ is BIBO.

$\therefore T_1$ is not $BIBO$ but T_2 is BIBO stable

Hence the correct option is (D).

Question 39 ***[Signals & Systems : Z-transform]***

If the Z-transform of a finite-duration discrete-time signal $x[n]$ is $X(z)$, then the Z - transform of the signal Z -transform of the signal $y[n]=x[2n]$ is :

(A) $Y(z)=X(z^2)$

(B) $Y(z)=\frac{1}{2}\left[X\left(z^{-1/2}\right)+X\left(-z^{-1/2}\right)\right]$

(C) $Y(z)=\frac{1}{2}\left[X\left(z^{1/2}\right)+X\left(-z^{1/2}\right)\right]$

(D) $Y(z)=\frac{1}{2}\left[X\left(z^{2}\right)+X\left(-z^{2}\right)\right]$

Ans. (C)

Sol. Given : If the z-Transform of finite duration signal.

$$x[n] \leftrightarrow x(z)$$

Let :

$$x[n]=\{\underset{\uparrow}{1}, 2, 3, 1\}$$

$$x(z) = \sum_{n=0}^{3} x(n)z^{-n}$$

$$x(z) = [1 + 2z^{-1} + 3z^{-2} + 1z^{-3}] \qquad \text{.... (i)}$$

From equation (i)

Replace z with the Square root of z.

$$x\left(z^{1/2}\right) = \left[1 + 2z^{-1/2} + 3z^{-2/2} + 1z^{-3/2}\right] \qquad \text{.... (ii)}$$

From Equation (i)

Replace z with $(-\sqrt{z})$.

$$x(-z^{1/2}) = [1 + 2(-1)^{-1} z^{-1/2} + 3(-1)^{-2} z^{-2/2} + 1(-1)^{-3} z^{-3/2}]$$

$$x(-z^{1/2}) = [1 + (-2) z^{-1/2} + 3z^{-2/2} + (-1) z^{-3/2}]$$

$$x(-z^{1/2}) = [1 + 2 z^{-1/2} + 3z^{-2/2} - 1 z^{-3/2}] \quad \ldots. \text{(iii)}$$

From Equation (ii) and (iii)

$$x(z^{1/2}) + x(-z^{-1/2}) = [2 + 6z^{-2/2}]$$

$$\frac{1}{2}[x(z^{1/2}) + x(-z^{-1/2})] = [1 + 3z^{-2/2}] \quad \ldots. \text{(iv)}$$

$$x(n) = \{1, 2, 3, 1\}$$

$$y(n) = x(2n) = \{1, 3\}$$

$$y(z) = 1 + 3z^{-1} \quad \ldots. \text{(v)}$$

From equation (iv) and (v)

$$Y(z) = \frac{1}{2}[x(z^{1/2}) + x(-z^{1/2})]$$

Hence the correct option is (C).

Question 40 ***[Electrical Machine : Synchronous Machine]***

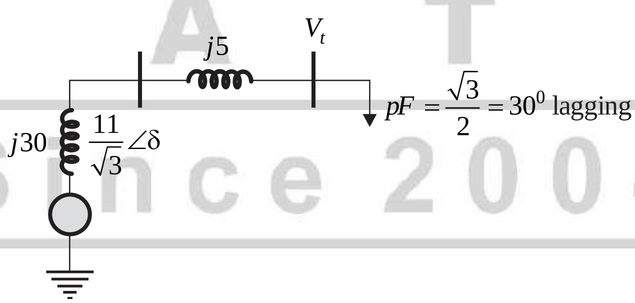
A 3-phase, 11 kV, 10 MVA synchronous generator is connected to an inductive load of power factor $(\sqrt{3}/2)$ via a lossless line with a per-phase inductive reactance of 5Ω. The per-phase synchronous reactance of the generator is 30Ω with negligible armature resistance. If the generator is producing the rated current at the rated voltage, then the power factor at the terminal of the generator is

(A) 0.63 lagging (B) 0.87 lagging

(C) 0.63 leading (D) 0.87 leading

Ans. (A)

Sol. Given circuit is shown below :



$$pF = \frac{\sqrt{3}}{2} = 30^0 \text{ lagging}$$

$$E = \sqrt{(V\cos 30 + IR)^2 + (V\sin 30^0 + IX)^2}$$

$$\frac{11K}{\sqrt{3}} = \sqrt{\left(V.\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{V}{2} + 524.86(5)\right)^2}$$

$$\left(\frac{11K}{\sqrt{3}}\right)^2 = V^2.\frac{3}{4} + \left(\frac{V}{2} + 2624.319\right)^2$$

$$\left(\frac{11K}{\sqrt{3}}\right)^2 = \frac{3}{4}V^2 + \frac{V^2}{4} + 2624.319\,\text{V} + 2624.319^2$$

$$V_t = 4618.105\,\text{V}$$

$$(IZ)^2 = E^2 + V^2 - 2EV\cos\delta$$

$$(2624.319)^2 = \left(\frac{11K}{\sqrt{3}}\right)^2 + (4618.105)^2 - 2\left(\frac{11K}{\sqrt{3}}\right)(4618.105)\cos\delta$$

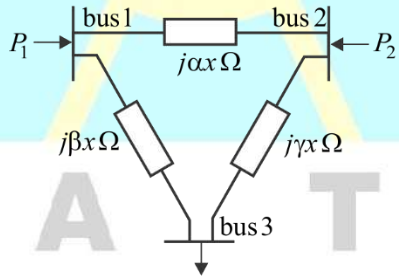
$$\delta = 20.96$$

$$pF = \cos(20.96 + 30) = 0.629\,\text{lagging}$$

Hence, the correct option is (A).

Question 41 ***[Power System : Power Flow]***

For the three-bus lossless power network shown in the figure, the voltage magnitudes at all the buses are equal to 1 per unit (pu), and the differences of the voltage phase angles are very small. The line reactances are marked in the figure, where $\alpha, \beta, \gamma,$ and x are strictly positive. The bus injections P_1 and P_2 are in pu. If $P_1 = mP_2$, where $m > 0$, and the real power flow from bus 1 to bus 2 is 0 pu, then which one of the following options is correct ?



(A) $\gamma = m\beta$ (B) $\beta = m\gamma$

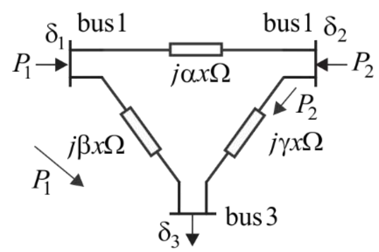
(C) $\alpha = m\gamma$ (D) $\alpha = m\beta$

Ans. (A)

Sol. Given :

(i) The voltage magnitudes at all the buses are equal to 1 per unit (pu) and the difference of the voltage phase angles are very small.

(ii) If $P_1 = mP_2$, where $m > 0$ and the real power flow from bus 1 to bus 2 is 0 pu.



From above diagram :

$$P_1 = \frac{|V_1||V_3|}{\beta x}\sin(\delta_1 - \delta_3)$$

$$P_1 = \frac{1}{\beta x}\sin(\delta_1 - \delta_2) \quad \text{.....(i)}$$

$$P_2 = \frac{|V_2||V_3|}{yx}\sin(\delta_2 - \delta_3) \quad \text{.....(ii)}$$

Given :

$$(\delta_1 - \delta_2) \approx (\delta_2 - \delta_3)$$

$$P_1 = mP_2$$

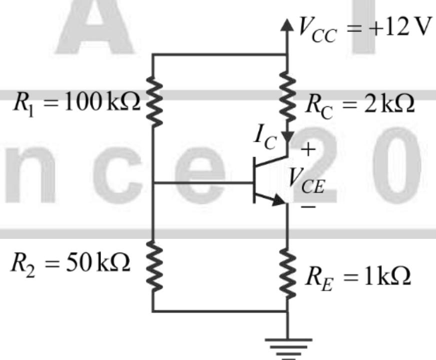
$$\frac{1}{\beta x}\sin(\delta_1 - \delta_2) = \frac{M}{yx}\sin(\delta_2 - \delta_3)$$

$$y = m\beta$$

Hence the correct option is (A).

Question 42 ***[Analog Electronics : BJT]***

A BJT biasing circuit is shown in the figure, where $V_{BE} = 0.7\,\text{V}$ and $\beta = 100$. The Quiescent Point values of V_{CE} and I_C are respectively :

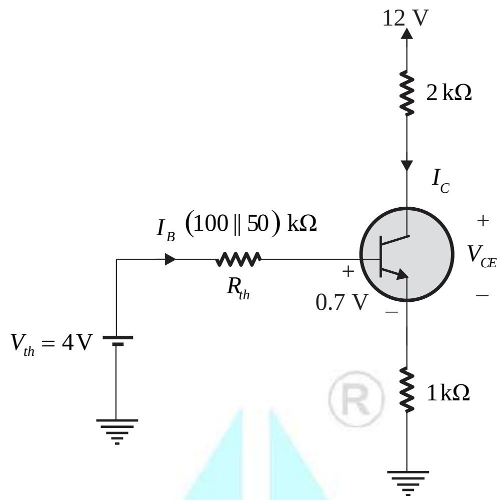


(A) $4.6\,\text{V}$ and $2.46\,\text{mA}$ (B) $3.5\,\text{V}$ and $2.46\,\text{mA}$

(C) $2.61\,\text{V}$ and $3.13\,\text{mA}$ (D) $4.6\,\text{V}$ and $3.13\,\text{mA}$

Ans. (A)

Sol.



$$V_{th} = \frac{12 \times 50}{100 + 50}$$

$$V_{th} = 4\text{V}$$

$$R_{th} = 50 \| 100 = 33.33 k\Omega$$

KVL : $-4 + I_B(33.33K) + 0.7 + (1+\beta)I_B \times 1k = 0$

$$I_B = \frac{3.3}{33.33k + 101k} = 24.56\,\mu\text{A}$$

$$I_C = \beta I_B = 100 \times 24.56 \times 10^{-6}$$

$$I_C = 2.46\,\text{mA}$$

$$I_E = (1+\beta) I_B = 2.47\,\text{mA}$$

KVL : $-12 + (2K)I_C + V_{CE} + I_E R_E = 0$

$$12 = 2K \times I_C + V_{CE} + I_E \times 1k$$

$$V_{CE} = 12 - (2k)(2.46\,\text{m}) - (2.47\,\text{m})(1k)$$

$$V_{CE} = 12 - 7.39 = 4.61\text{V}$$

$$V_{CE} = 4.61\text{V}$$

Hence the correct option is (A).

Question 43 [MSQ] **[Mathematics : Maxima Minima]**

Let $f(t)$ be a real-valued function whose second derivative is positive for $-\infty < t < \infty$. Which of the following is/are always true ? **(2 Marks)**

(A) $f(t)$ has at least one local minimum

(B) $f(t)$ cannot have two distinct local minima

(C) $f(t)$ has at least one local maximum

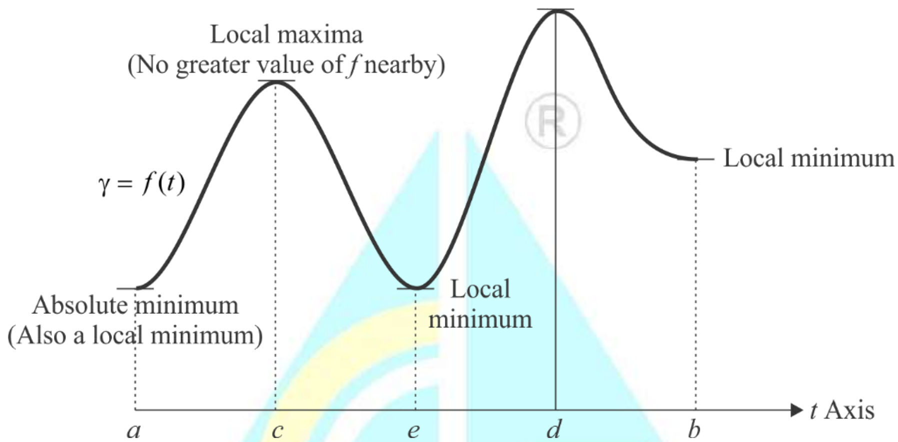
(D) The minimum value of $f(t)$ cannot be negative

Ans. (B)

Sol. **Given** : $f(t)$ is a real value function whose second derivative is positive for $-\infty < t < \infty$.

Suppose $f(t)$

Absolute maxima (Global maximum) (Also a local maxima)



From option (A) :

This is correct statement. $f(t)$ has at least one local minimum.

From option (B) :

This is wrong statement. Because $f(t)$ can have two distinct local minima.

From option (C) :

This is wrong statement. Because according to question second derivative is positive for $-\infty < t < \infty$. Therefore, there will not be local maxima.

Key Point

The stationary points for finding the extreme value of a function $f(t)$ are given by $f'(t) = 0$

$(\bullet)$ condition for maxima : $f''(t) < 0$

$(\bullet)$ condition for minima : $f''(t) > 0$

From option (D) :

This is wrong statement. The minimum value of $f(t)$ can be negative. Hence the correct option is (B).

Question 44 [MSQ] **[Mathematics : Limit Series]**

Consider the function $f(t) = (\max(0,t))^2$ for $-\infty < t < \infty$, where $\max(a,b)$ denotes the maximum of a and b. Which of the following statements is/are true ? **(2 Marks)**

(A) $f(t)$ is not differentiable

(B) $f(t)$ is differentiable and its derivative is continuous

(C) $f(t)$ is differentiable but its derivative is not continuous

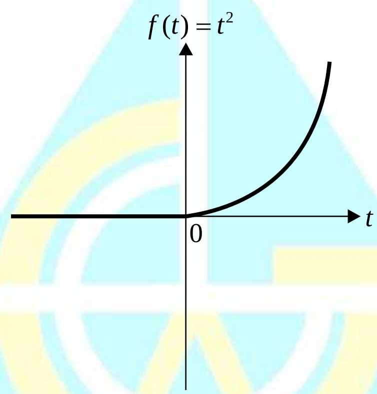
(D) $f(t)$ and its derivative are differentiable

Ans. **(B)**

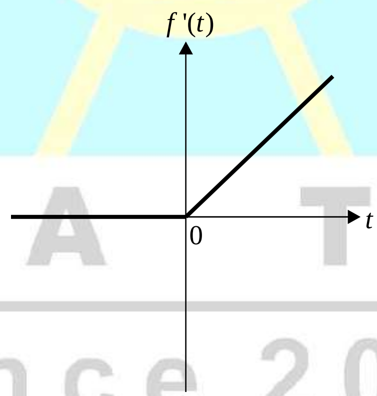
Sol. **Given :** $f(t)=\left[\max(0,t)\right]^2$ For, $-\infty<t<\infty$

$$f(t)=\begin{cases} t^2 & t\geq 0 \\ 0 & t<0 \end{cases}$$

(i) $f(t)=t^2$



(ii) $f'(t)=2t$



$\Rightarrow f'(t)$ exists. But this is not differentiable.

$\Rightarrow f''(t)$ does not exist.

$\Rightarrow f(t)$ is differentiable.

$\therefore$ $f(t)$ is differentiable and $f'(t)$ is continuous. But $f''(t)$ is not continuous.

Hence the correct option is (B).

Question 45 [MSQ] ***[Mathematics : Differential Equation]***

Which of the following differential equations is/are nonlinear ? **(2 Marks)**

(A) $tx(t)+\frac{dx(t)}{dt}=t^2e^t, \quad x(0)=0$

(B) $\frac{1}{2}e^t+x(t)\frac{dx(t)}{dt}=0, \quad x(0)=0$

(C) $x(t)\cos t-\frac{dx(t)}{dt}\sin t=1, \quad x(0)=0$

(D) $x(t)+e^{\left(\frac{dx(t)}{dt}\right)}=1, \quad x(0)=0$

Ans. (B, D)

Sol.

Key Point

A differential equation is non-linear differential equation if :

(1) Its degree is more than one.

(2) Any one of differential coefficients has order more than one.

(3) Products containing dependent variable and its differential coefficient are present.

For linear DE :

$$P_0\frac{d^nx(t)}{dt^n}+P_1\frac{d^{n-1}x(t)}{dt^{n-1}}+.......P_{n-1}\frac{dx(t)}{dt}+P_nx(t)=Q$$

Condition for linear DE :

(1) $P_0, P_1, ...P_n$ and Q are either constant or functions of independent variable t.

(2) The function $x(t)$ and its derivatives occurs in the equation up to the first degree only.

(3) No product of $x(t)$ and/or any of its derivatives are present.

(4) No transcendental functions – (Trigonometric or logarithmic etc.) of x or any of its derivatives occur.

From option (A) :

$$tx(t)+\frac{dx(t)}{dt}=t^2e^t \qquad x(0)=0$$

$$\Rightarrow tx(t)+\frac{dx(t)}{dt}=t^2e^t \qquad x(0)=0$$

This is a linear differential equation because it satisfies all the conditions of Linear differential equation.

From option (B) :

$$\frac{1}{2}e^t+x(t)\frac{dx(t)}{dt}=0$$

This is non linear differential equation because of the presence of product of dependent variable and its derivative.

From option (C) :

$$x(t)\cos t-\frac{dx(t)}{dt}\sin t=1 \qquad x(0)=0$$

This is a linear differential equation because it satisfies all the conditions of Linear differential equation.

From option (D) :

$$x(t)+e^{\left(\frac{dx(t)}{dt}\right)}=1$$

$$e^{\left(\frac{dx}{dt}\right)}=1+\frac{\left(\frac{dx}{dt}\right)}{\lfloor 1}+\frac{\left(\frac{dx}{dt}\right)^2}{\lfloor 2}+\frac{\left(\frac{dx}{dt}\right)^3}{\lfloor 3}+.......$$

This is a non linear differential equation because of the presence of derivative of transcendental function of independent variable *x*.

Hence, the correct options are (B) and (D).

Question 46 **[MSQ]** ***[Electrical Machine : Synchronous Machine]***

For a two-phase network, the phase voltages V_p and V_q are to be expressed in terms of sequence voltages V_α and V_β as $\begin{bmatrix}V_p\\V_q\end{bmatrix}=S\begin{bmatrix}V_\alpha\\V_\beta\end{bmatrix}$. The possible option(s) for matrix S is/are :

(A) $\begin{bmatrix}1&1\\1&-1\end{bmatrix}$

(B) $\begin{bmatrix}1&1\\1&1\end{bmatrix}$

(C) $\begin{bmatrix}1&1\\1&0\end{bmatrix}$

(D) $\begin{bmatrix}-1&1\\1&1\end{bmatrix}$

Ans. (A, D)

Question 47 **[MSQ]**

Which of the following options is/are correct for the Automatic Generation Control (AGC) and Automatic Voltage Regulator (AVR) installed with synchronous generators ?

(A) AGC response has a local effect on frequency while AVR response has a global effect on voltage.

(B) AGC response has a global effect on frequency while AVR response has a local effect on voltage.

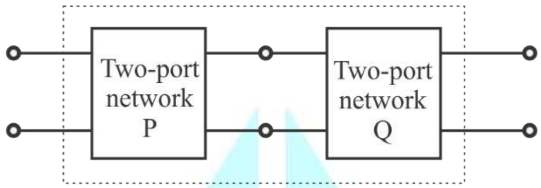
(C) AGC regulates the field current of the synchronous generator while AVR regulates the generator's mechanical power input.

(D) AGC regulates the generator's mechanical power input while AVR regulates the field current of the synchronous generator.

Ans. (B, D)

Question 48 ***[Network Theory : Two Port Network]***

Two passive two-part networks P and Q are connected as shown in the figure. The impedance matrix of network P is $Z_P = \begin{bmatrix} 40\,\Omega & 60\,\Omega \\ 80\,\Omega & 100\,\Omega \end{bmatrix}$. The admittance matrix of network Q is $Y_Q = \begin{bmatrix} 5\,S & -2.5\,S \\ -2.5\,S & 1\,S \end{bmatrix}$. Let the $ABCD$ matrix of the two-part network R in the figure be $\begin{bmatrix} \alpha & \beta \\ \gamma & \delta \end{bmatrix}$. The value of β in Ω is _______ (Rounded off to 2 decimal places).



Two-port network R

Ans. **– 9.90 to – 19.70**

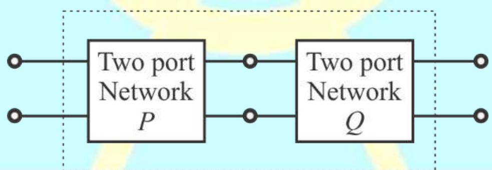
Sol. Given Z parameter matrix of network P is

$$Z_P = \begin{bmatrix} 40\,\Omega & 60\,\Omega \\ 80\,\Omega & 100\,\Omega \end{bmatrix}$$

And the admittance matrix of network Q is

$$Y_Q = \begin{bmatrix} 5s & -2.5s \\ -2.5s & 1s \end{bmatrix}$$

Given circuit is shown below



In case of cascade connection of two - port network individual ABCD parameter are multiplied.

Standard z- parameter equation :

$$V_1 = Z_{11}I_1 + Z_{12}I_2$$

$$V_2 = Z_{21}I_1 + Z_{22}I_2$$

Standard ABCD parameter equation

$$V_1 = AV_2 - BI_2$$

$$I_1 = CV_2 - DI_2$$

Now according to question

$$V_1 = 40I_1 + 60I_2 \qquad \ldots\text{(i)}$$

$$V_2 = 80I_1 + 100I_2 \qquad \ldots\text{(ii)}$$

From equation (ii)

$$80I_1 = V_2 - 100I_2$$

$$I_1 = \frac{V_2}{80} - \frac{100}{80} I_2 \qquad \ldots\text{(iii)}$$

Put the value of I_1 in equation (i)

$$V_1 = 40\left(\frac{V_2}{80} - \frac{100}{80} I_2\right) + 60I_2$$

$$V_1 = \frac{V_2}{2} - 50I_2 + 60I_2$$

$$V_1 = \frac{V_2}{2} + 10I_2$$

$$V_1 = \frac{V_2}{2} - (-10)I_2 \qquad \ldots\text{(iv)}$$

From equation (iii) and (iv)

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_P = \begin{bmatrix} \frac{1}{2} & -10 \\ \frac{1}{80} & \frac{100}{80} \end{bmatrix}$$

Now,

Standard Y - parameter equation

$$I_1 = Y_{11} V_1 + Y_{12} V_2$$

$$I_2 = Y_{21} V_1 + Y_{22} V_2$$

According to question Y – Parameter equation :

$$I_1 = 5V_1 - 2.5V_2 \qquad \ldots\text{(v)}$$

$$I_2 = -2.5V_1 + V_2 \qquad \ldots\text{(vi)}$$

From equation (vi) :

$$2.5V_1 = V_2 - I_2$$

$$V_1 = \frac{V_2}{2.5} - \frac{I_2}{2.5} \qquad \ldots\text{(vii)}$$

Put the value of V_1 in equation 5

$$I_1 = 5\left[\frac{V_2}{2.5} - \frac{I_2}{2.5}\right] - 2.5V_2$$

$$I_1 = \frac{5}{2.5} V_2 - 2.5V_2 - \frac{5}{2.5} I_2$$

$$I_1 = V_2\left(2 - \frac{5}{2.5}\right) - 2I_2$$

$$I_1 = \left(-\frac{1}{2}\right)V_2 - 2I_2 \qquad \text{....(viii)}$$

From equation (vii) and (viii)

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_Q = \begin{bmatrix} \frac{1}{2.5} & \frac{1}{2.5} \\ \left(-\frac{1}{2}\right) & 2 \end{bmatrix}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}_P \begin{bmatrix} A & B \\ C & D \end{bmatrix}_Q$$

$$= \begin{bmatrix} \frac{1}{2} & -10 \\ \frac{1}{80} & \frac{10}{8} \end{bmatrix} \begin{bmatrix} \frac{2}{5} & \frac{2}{5} \\ -\frac{1}{2} & 2 \end{bmatrix}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \left(\frac{1}{2}\times\frac{2}{5}+10\times\frac{1}{2}\right) & \left(\frac{1}{2}\times\frac{2}{5}-20\right) \\ \left(\frac{2}{5}\times\frac{1}{80}-\frac{10}{8}\times\frac{1}{2}\right) & \left(\frac{1}{80}\times\frac{2}{5}-\frac{10}{8}\times 2\right) \end{bmatrix}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \left(\frac{1}{5}+5\right) & \left(\frac{1}{5}-20\right) \\ \frac{1}{200} & \left(\frac{1}{200}-\frac{10}{4}\right) \end{bmatrix}$$

$$= \begin{bmatrix} 5.2 & -19.8 \\ 0.005 & -2.495 \end{bmatrix}$$

After comparing

$\alpha = 5.2$

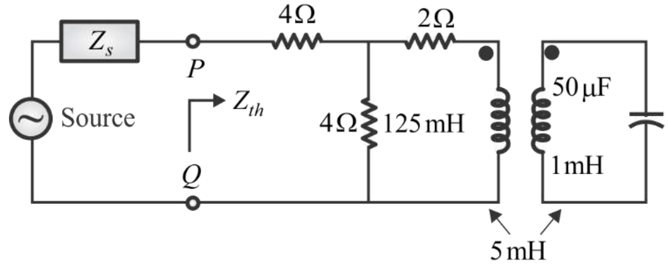
$\beta = -19.8\Omega$

$\gamma = 0.005℧$

$\delta = -2.495$

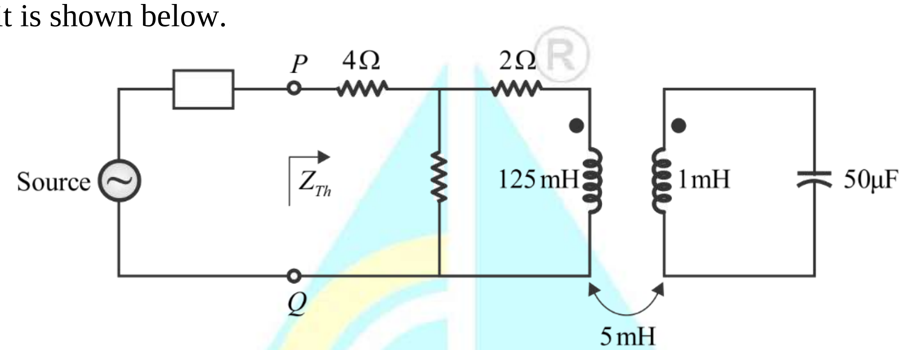
Question 49 ***[Network Theory : Magnetic Coupled circuit]***

For the circuit shown in the figure, the source frequency is 5000 rad/sec. the mutual inductance between the magnetically coupled inductors is 5 mH with their self inductances being 125 mH and 1 mH. The venin's impedence, $Z_{th,}$ between the terminals P and Q in Ω is __________ (rounded off to 2 decimal places).



Ans. **5.32 to 5.34**

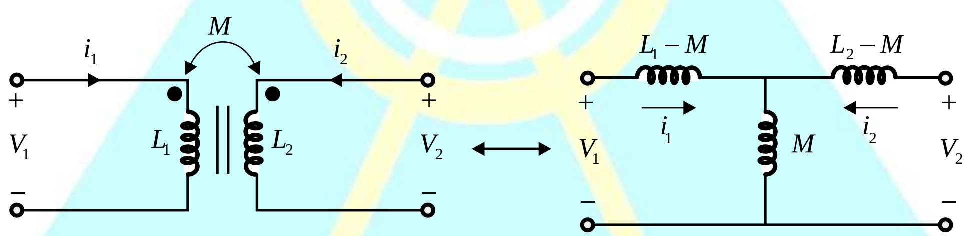
Sol. Given Circuit is shown below.



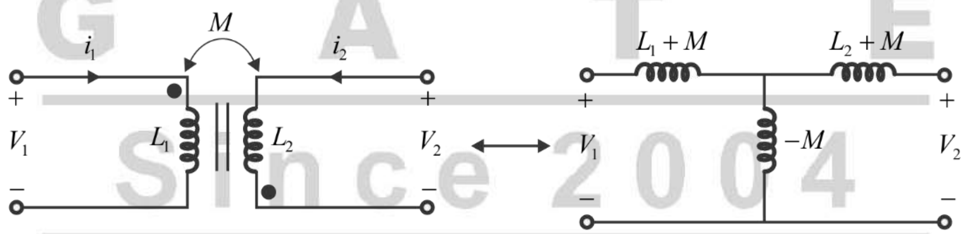
Source frequency $= 5000$ rad/sec

Important points to Remember :

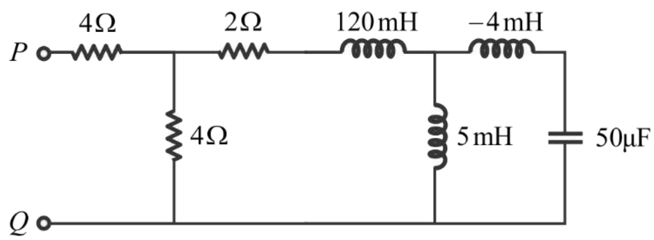
(·) Magnetic aiding :



(·) Magnetic opposition :



Step 1 :- Convert are the values of inductor and capacitor into ohmic value.



$$\Rightarrow \quad X_{L_1} = j\omega L_1 = j5000\times 120\times 10^{-3}$$

$$X_{L_1} = j600\ \Omega$$

$$\Rightarrow \quad X_{L_2} = j\omega L_2 = j5000 \times (-4) \times 10^{-3}$$

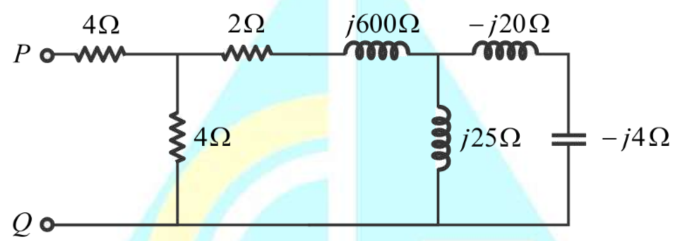
$$X_{L_2} = -j20\ \Omega$$

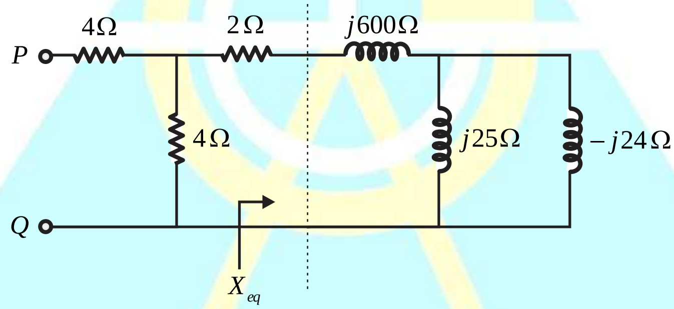
$$\Rightarrow \quad X_m = j\omega M = j(5000)(5) \times 10^{-3}$$

$$X_m = j25\ \Omega$$

$$\Rightarrow \quad X_c = \frac{-j}{\omega c} = \frac{-j}{(5000)(50) \times 10^{-6}}$$

$$X_c = -j4\ \Omega$$





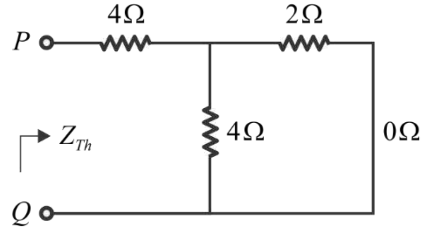
$$X_{eq} = j600 + \left(\frac{-j25 \times j24}{-j24 + j25}\right)$$

$$X_{eq} = j600 + \frac{600}{j} = 0\ \Omega$$

$$X_C = \frac{-j}{\omega c} = \frac{j}{5000 \times 50 \times 10^{-6}}$$

$$X_C = -j4\Omega$$

Now,



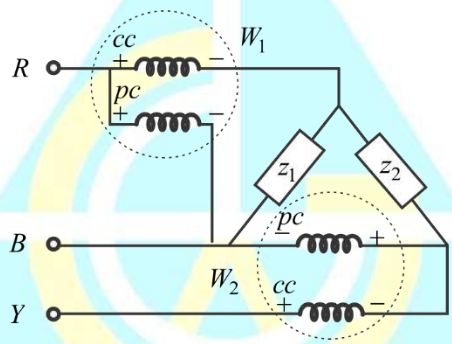
$$Z_{Th} = 4 + \left(\frac{4\times 2}{6}\right)$$

$$Z_{Th} = 4 + \frac{8}{6} = \frac{24+8}{6}$$

$$Z_{Th} = \frac{32}{6} = 5.33\Omega$$

Question 50 ***[Measurement : Watt Meter]***

In the circuit shown, $Z_1 = 50\angle -90^0\,\Omega$ and $Z_2 = 200\angle -30^0\,\Omega$. It is supplied by a three phase 400 V source with the phase sequence being R-Y-B. Assume the watt meters W_1 and W_2 to be ideal. The magnitude of the difference between the readings of W_1 and W_2 in watts is ________ (rounded off to 2 decimal places).



Ans. 692 to 693

Sol. **Given :**

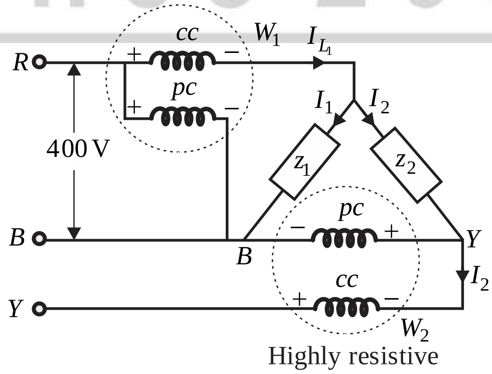
(i) $Z_1 = 50\angle -90^0\,\Omega$

(ii) $Z_2 = 200\angle -30^0\,\Omega$

(iii) Line Voltage = 400 V

(iv) Assume the wattmeter W_1 and W_2 to be Ideal.

Given circuit is shown below:



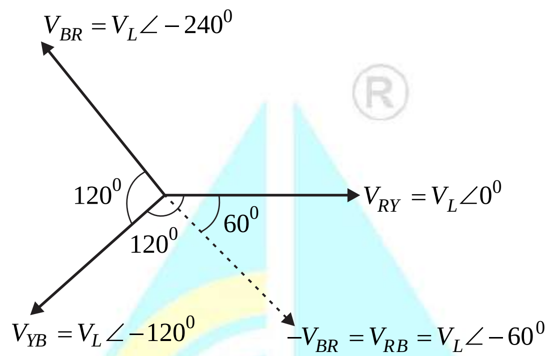
$V_{RY} = 400\angle 0^0$

$V_{YB} = 400\angle -120^0$

$V_{BR} = 400\angle -240^0$ [$\therefore$ According to given phase sequence]

Phasor diagram : ($R - Y - B$ phase sequence)

$V_{BR} = 400\angle -240^0$



$\therefore \quad V_{BR} = V_{BN} - V_{RN}$

$-V_{BR} = V_{RN} - V_{BN} = V_{RB}$

$[-V_{BR} = V_{RB}]$

From Circuit diagram:

$$I_1 = \frac{V_{RB}}{Z_1} = \frac{400\angle -60^0}{50\angle -90^0} = 8\angle 30^0 \quad \text{Amp}$$

$$I_2 = \frac{V_{RY}}{Z_2} = \frac{400\angle 0^0}{200\angle -30^0} = 2\angle 30^0 \quad \text{Amp}$$

Wattmeter (W_1) Reading:

$W_1 = V_{RB} I_{L_1} \cos\left(\angle V_{RB} \text{ and } I_{L_1}\right)$

$I_{L_1} = I_1 + I_2$

$= 8\angle 30^0 + 2\angle 30^0$

$I_{L_1} = 10\angle 30^0$ Amp

$W_1 = 400 \times 10\cos(\theta_v - \theta_i)$

$W_1 = 400 \times 10 \times \cos(-60 - 30^0)$

$W_1 = 400 \times 10\cos(90^0) = 0$ Watt

Wattmeter (W_2) Reading:

$W_2 = V_{YB}\, I_2 \cos\left(\angle V_{YB} \text{ and } I_2\right)$

$W_2 = 400 \times 2\cos(-120 - 30^0)$

$$W_1 = 800\cos(150^0) = (-800)\cos(30^0)$$

$$W_1 = -800\left(\frac{\sqrt{3}}{2}\right) = -\,692.82 \text{ Watt}$$

The magnitude of the difference between the readings of W_1 and W_2 in watts

$$|w_1 - w_2| = |0 - (-692.82)|$$

$$|w_1 - w_2| = 692.82 \text{ watts}$$

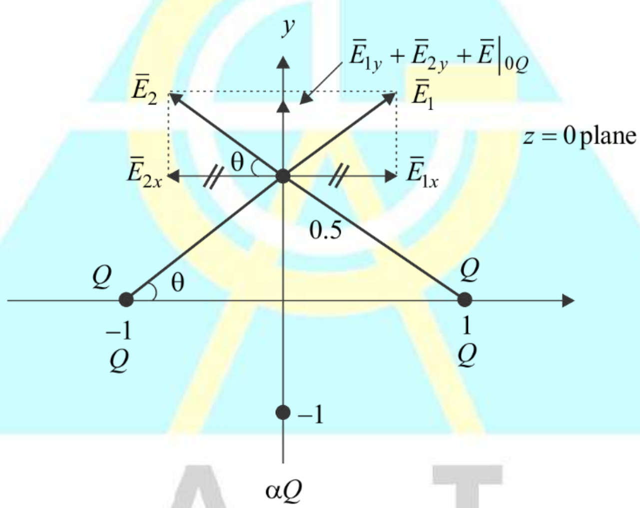
Hence the correct answer is 692.82.

Question 51 ***[EMT : Electrostatic]***

In the (x, y, z) coordinate system, three point-charges $Q, Q,$ and αQ are located in free space at $(-1, 0, 0), (1, 0, 0),$ and $(0, -1, 0)$, respectively. The value of α for the electric field to be zero at $(0, 0.5, 0)$ is ________ (rounded off to 1 decimal place).

Ans. **– 1.7 to – 1.5**

Sol.



From the figure,

$$\bar{E}_{net} = \bar{E}_{1y} + \bar{E}_{2y} + \bar{E}\big|_{\alpha Q} = 0 \qquad \text{[as per question]}$$

Here, $\bar{E}_{1y} = \bar{E}_{2y} = \bar{E}\big|_Q$ (say)

$\therefore \quad \bar{E}_{net} = 2\bar{E}\big|_Q + 2\bar{E}\big|_{\alpha Q} = 0$

Now, $\bar{E}\big|_Q = \dfrac{kQ}{\left[\sqrt{1+0.5^2}\right]^2}\sin\theta\,\hat{a}_y; \qquad k = \dfrac{1}{4\pi\epsilon_0}$

$$\bar{E}\big|_Q = \frac{kQ}{1.25}\times\frac{0.5}{\sqrt{1+0.5^2}}\hat{a}_y = 0.357\,kQ\hat{a}_y$$

Also, $\bar{E}\big|_{\alpha Q} = \dfrac{k(\alpha Q)}{(1.5)^2}\hat{a}_y = 0.444k(\alpha Q)\hat{a}_y$

Hence, $\bar{E}_{net} = 2\times 0.357kQ\hat{a}_y + 0.444k(\alpha Q)\hat{a}_y = 0$

$$0.715kQ = -0.444k(\alpha Q)$$

$$\alpha = \frac{-0.715}{0.444} = -1.61$$

$\therefore \quad \alpha = -1.61$

Question 52

The given equation represents a magnetic field strength $\bar{\text{H}}(r,\theta,\phi)$ in the spherical coordinate system, in free space. Here, $\hat{r}$ and $\hat{\theta}$ represent the unit vectors along r and θ, respectively. The value of P in the equation should be __________ (rounded off to the nearest integer).

$$\vec{H}(r,\theta,\phi) = \frac{1}{r^3}\left(\hat{r}P\cos\theta + \hat{\theta}\sin\theta\right)$$

Ans. 2 to 2

Question 53 ***[Signals & Systems]***

If the energy of a continuous-time signal $x(t)$ is E and the energy of the signal $2x(2t-1)$ is cE, then c is _______ (rounded off to 1 decimal place).

Ans. 2.0 to 2.0

Sol. It the Energy of a Continuous time signal $x(t)$ is E.

(i) $x(t) \longrightarrow \text{Energy}(E)$

(ii) $x(\text{at})\big|_{a\neq 0} \longrightarrow \dfrac{E}{|a|}$

(iii) $kx(\text{at}) \longrightarrow \dfrac{k^2E}{|a|}$

(iv) $x(t-t_0) \longrightarrow E$

Now According to Given signal:

$$2x(2t-1) \longrightarrow CE \qquad \ldots\text{(i)}$$

$$x(t-1) \longrightarrow E$$

$$x(2t-1) \longrightarrow \frac{E}{|2|} = \frac{E}{2}$$

$$2x(2t-1) \longrightarrow (2)^2\frac{E}{2} = 2E \qquad \ldots\text{(ii)}$$

After comparing we get

$C = 2$

Hence the Correct Answer is (2)

Question 54 ***[Electrical Machine : Induction Machine]***

A 3-phase star connected slip ring induction motor has the following parameters referred to the stator :

$$R_s = 3\Omega, X_s = 2\Omega, X_r^{'} = 2\Omega, R_r^{'} = 2.5\Omega$$

The per phase stator to rotor effective turns ratio is $3:1$. The rotor winding is also star connected. The magnetizing reactance and core loss of the motor can be neglected. To have maximum torque at starting, the value of the extra resistance in ohms (effective to the rotor side) to be connected in series with each phase of the rotor winding is _______ (rounded off to 2 decimal places).

Ans. **0.26 to 0.29**

Sol. A 3-phase star connected slip ring Induction Motor.

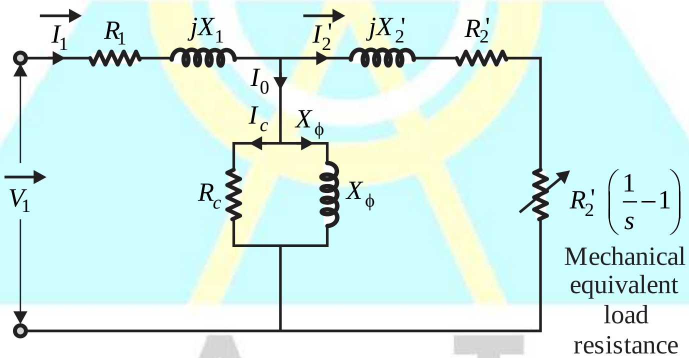
(i) $R_s = 3\Omega$, $X_s = 2\Omega$

(ii) $X_r^{'} = 2\Omega$, $R_r^{'} = 2.5\Omega$

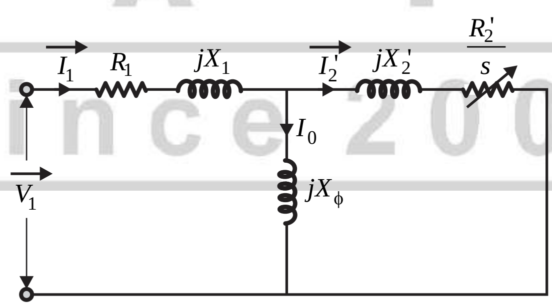
(iii) The per phase stator to rotor effective turns ratio is $3:1$

(iv) The magnetizing reactance and core Loss of the Motor can be neglected.

Exact Equivalent circuit referred to stator side.



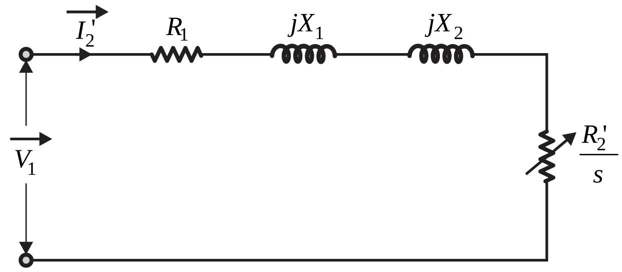
- IEEE Approved (Steinmetz model]



Now, according to question :

- Magnetizing reactance can be neglected.

  Redraw above circuit diagram :



Condition for the maximum Torque:

$$\frac{R_2'}{s} = \sqrt{(R_1)^2 + (x_1 + x_2)^2} = \sqrt{9+16}$$

$$\frac{R_2'}{s} = \sqrt{25} = 5$$

$$\frac{R_2'}{s} = 5 \qquad \text{.... (i)}$$

🕮 Key Point

1. At No Load, the rotor speed is very close to the Synchronous speed. So the slip. In this case is very less (or) near zero
$$s \approx 0$$
2. $\text{Slip}(s) = \frac{N_s - N_r}{N_s}$
3. As load Increase the rotor speed decrease and increasing the slip.
4. Slip of an induction motor is 1 at the start and it decrease as rotor gaining speed.
5. In the case of slip ring induction motor the rotor resistance is increased by the addition of external resistance. This External resistance makes the rotor circuit current (I_2) Low. Hence by increasing the rotor resistance we can improve the starting. Torque of the wound rotor induction Motor (SRIM).
6. In squirrel cage type. External resistance addition is not possible.

From equation (i)

$$\frac{R_2' + R_{ext}}{S} = 5$$

To Have maximum Torque at starting

$$S = 1$$

$$R_2' + R_{\text{ext}} = 5$$

$$R_{\text{ext}} = (5 - 2.5)$$

$$R_{\text{ext}} = 2.5\ \Omega$$

Rext Referred to rotor side is $= \left(\frac{1}{3}\right)^2$ Rest

$$= \frac{1}{9} \times 2.5 = 0.277\Omega$$

Hence the correct answer is (0.277).

Question 55 **[Electrical Machine : DC Machine]**

A 5 kW, 220 V DC shunt motor has 0.5Ω armature resistance including brushes. The motor draws a no-load current of 3 A. The field current is constant at 1 A. assuming that the core and rotational losses are constant and independent of the load, the current (in amperes) drawn by the motor while delivering the rated load, for the best possible efficiency, is _______ (rounded off to 2 decimal places).

Ans. **27.19 to 27.39**

Sol. **Given :**

(i) $V_t = 220\,\text{V}$

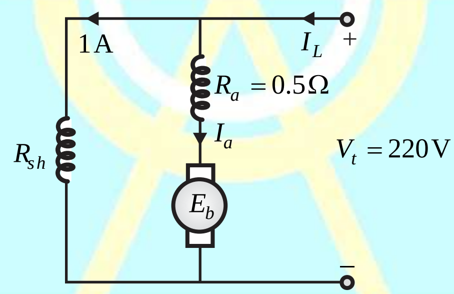
(ii) $R_a = 0.5\Omega$

(iii) Field current $(I_{sn}) = 1\text{A}$

(iv) Output power $= 5\,\text{kW}$

(v) Assuming that the core and rotational losses are constant and independent of the load.

(vi) $I_L(\text{No} - \text{Load}) = 3\text{A}$



Case 1 : At no load :

$$E_{b_1} = V_t - I_a R_a$$

$$E_{b_1} = 220 - (3-1)0.5$$

$$E_{b_1} = 219\,\text{Volt}$$

No load power developed = Mechanical loss

$$= E_{b_1} I_a$$

$$= 219 \times 2 = 438\,\text{W}$$

Case 2 : At rated load :

Given :

$$P_{out} = 5\,\text{kW}$$

Mechanical power developed = Net mechanical power output + Mechanical losses in armature $=\left(E_b I_a\right)$

$$E_{b_2} I_{a_2} = 5000 + 438 = 5438\,\text{W}$$

$$\left(V - I_{a_2} R_a\right) I_{a_2} = 5438$$

$$V I_{a_2} - I_{a_2}^2 R_a = 5438$$

$$(0.5) I_{a_2}^2 - (220) I_{a_2} + 5438 = 0$$

$$I_{a_2} = 26.28 \text{ Amp}$$

Current drawn by motor

$$I_L = I_{a_2} + I_{sh}$$

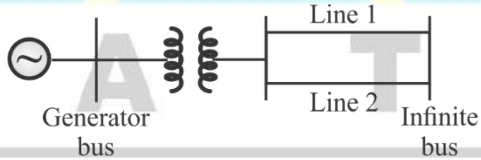
$$I_L = 26.28 + 1$$

$$I_L = 27.28\,\text{Amp}$$

Hence the correct Answer is (27.28)

Question 56 ***[Power System : PSM Stability]***

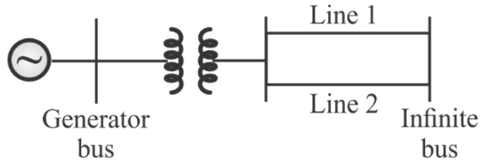
The signal line diagram of a lossless system is shown in the figure. The system is operating in steady-state at a stable equilibrium point with the power output of the generator being $P_{\max} \sin\delta$, where δ is the load angle and the mechanical power input is $0.5P_{\max}$. A fault occurs on line 2 such that the power output of the generator is less than $0.5P_{\max}$ during the fault. After the fault is cleared by opening line 2, the power output of the generator is $\left\{P_{\max}/\sqrt{2}\right\}\sin\delta$. If the critical fault clearing angle is $\pi/2$ radians, the accelerating area on the power angle curve is ________ times $P_{\max}$ (rounded off t 2 decimal places).



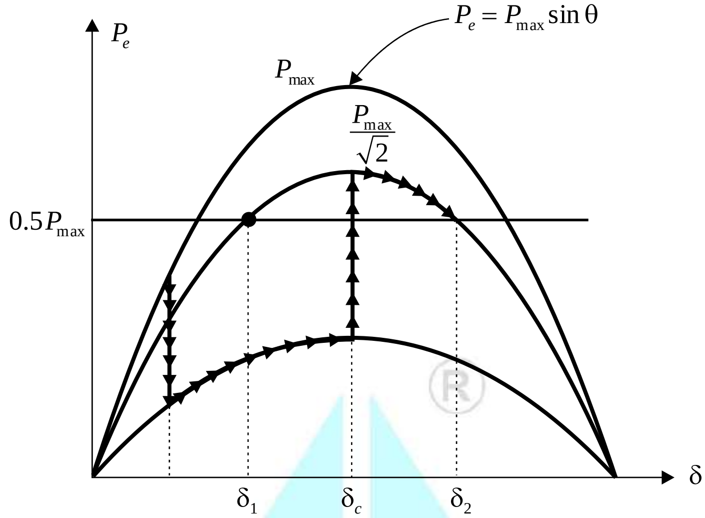
Ans. 0.10 to 0.12

Sol. Given:

(i) Power System Network :



(ii) The mechnical input power $= 0.5\,P_{\max}$



$$P_e = \frac{P_{\max}}{\sqrt{2}}\sin\delta_1$$

$$0.5\,P_{\max} = \frac{P_{\max}}{\sqrt{2}}\sin\delta_1$$

$$\delta_1 = \sin^{-1}\left(\frac{0.5P_{\max}}{\frac{P_{\max}}{\sqrt{2}}}\right)$$

$$\delta_1 = \sin^{-1}(0.5\times\sqrt{2})$$

$$\delta_1 = \sin^{-1}(0.707)$$

$$\delta_1 = 45^0$$

Now

$$\delta_2 = 180^0 - \delta_1 = 180 - 45$$

$$\delta_2 = 135^0$$

For critical condition:

Accelerating Area = Deaccelerating Area

$$A_1 = A_2$$

$$= \int_{90^\circ}^{135^\circ}\left(\frac{P_{\max}}{\sqrt{2}}\sin\delta - 0.5P_{\max}\right)d\delta$$

$$= P_{\max}\left[\frac{1}{\sqrt{2}}(-\cos\delta) - 0.5(\delta)\right]_{90^\circ}^{135^\circ}$$

$$= P_{\max}\left[\frac{1}{\sqrt{2}}(\cos(135)) - 0.5(135-90)\frac{\pi}{180}\right]$$

$$= 0.107P_{\max}$$

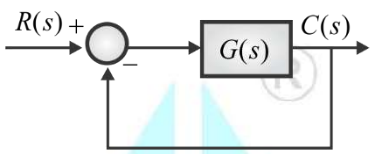
Hence, the correct answer is 0.107.

Question 57 ***[Control System : Root Locus]***

Consider the closed-loop system shown in the figure with

$$G(s)=\frac{K\left(s^2-2s+2\right)}{\left(s^2+2s+5\right)}$$

The root locus for the closed-loop system is to be drawn for $0\le K\le\infty$. The angle of departure (between 0^0 and 360^0) of the root locus branch drawn from the pole $(-1+j2)$, in degrees, is _________ (rounded off to the nearest integer).



Ans. **4 to 8**

Sol. Given : $G(s)=\dfrac{k\left(s^2-2s+2\right)}{\left(s^2+2s+5\right)}$

(i) Number of poles and zeros :

Number of zeros = 2

Number of poles = 2

(ii) Location of poles and zeros :

Location of zeros $s=\dfrac{2\pm\sqrt{4-8}}{2}$

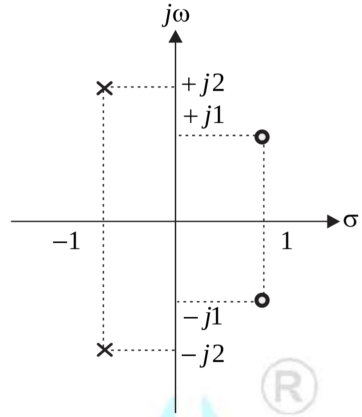
$$s=\frac{2\pm 2i}{2}=(1\pm i)$$

Location of poles

$$s=\frac{-2\pm\sqrt{4-20}}{2}$$

$$s=\frac{-2\pm 4i}{2}=(-1\pm 2i)$$

(iii) Pole-zero diagram :



(iv) No. of poles = No. of zeros.

$P = Z$ (Improper Transfer Function)

(v) Number of root locus branch :

$B = P = Z$ (If P = Z)

Note : Number of root locus branch

$= \max(P, Z)$

(vi) Number of asymptotes :

$A = 0$ (If P = Z)

(vii) Angle of asymptotes :

For $P = Z$. There are no asymptotes.

(viii) Centroid (σ): For $P = Z$,

There are no centroid.

(ix) Angle of departure : Angle of departure is valid for complex conjugate poles.

Angle of departure

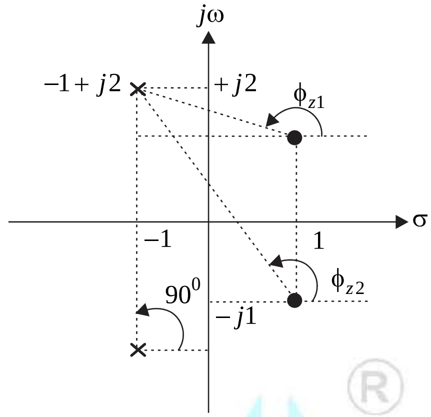
$(\phi)_d = 180 - \phi$

where

$\phi = \sum \phi_p - \sum \phi_z$

$\sum \phi_p =$ is the sum of all angles contributed by remaining poles.

$\sum \phi_z =$ is the sum of all angles contributed by all the zeros.



$$\sum \phi_p = 90^0$$

$$\phi_{z_1} = 180 - \tan^{-1}\left(\frac{1}{2}\right) = 153.44^0$$

$$\phi_{z_2} = 180 - \tan^{-1}\left(\frac{3}{2}\right) = 123.7^0$$

$$\sum \phi_z = 153.44^0 + 123.7^0$$

$$\sum \phi_z = 277.14^0$$

$$\phi = \sum \phi_p - \sum \phi_z$$

$$\phi = 90^0 - 277.14^0$$

$$\phi = -187.14^0$$

Angle of departure

$$\phi_d = 180 - \phi$$

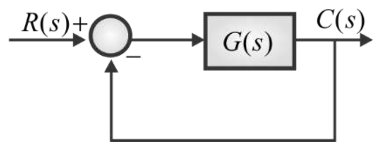
$$\phi_d = 180 - \left(-187.14^0\right)$$

$$\phi_{d(-1+j2)} = 367.14^0 \text{ (or) } 7.14^0$$

Hence the correct answer is 7.

Question 58 ***[Control System : Bode Plot]***

Consider the stable closed-loop system shown in the figure. The asymptotic Bode magnitude plot of $G(s)$ has a constant slope of -20 dB / decade at least till 100 rad / sec with the gain crossover frequency being 10 rad / sec. The asymptotic Bode phase plot remains constant at -90^0 at least till $\omega = 10$ rad / sec. The steady-state error of the closed-loop system for a unit ramp input is _________ (rounded off to 2 decimal places).

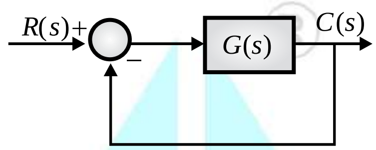


Ans. 0.09 to 0.11

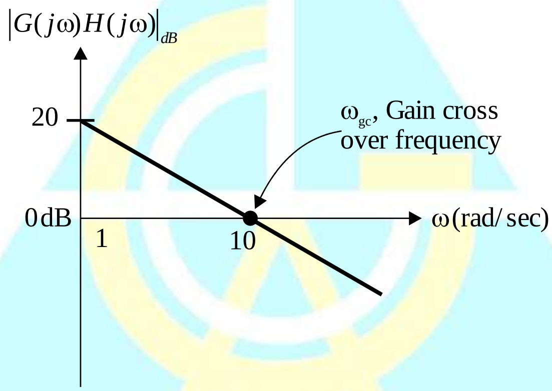
Sol. **Given :** Initial slope = – 20 dB/dec

Method 1

- $$R(s) = \frac{1}{s^2}$$



According to Question:



The Initial slope is (-20 dB/dec) and this corresponds to a factor S in the denominator of transfer function (one pole at origin)

Open Lop Transfer function $= \dfrac{k}{s}$

$$20\log_{10} k - 20\log_{10}(1) = 20$$

$$20\log_{10} k = 20$$

$$\log_{10} k = 1$$

$$k = 10$$

- **Steady state Error of the closed loop system for a unit ramp input.**

$$e_{ss} = \lim_{s\to 0}\frac{sR(s)}{1+G(s)H(s)}$$

$$e_{ss} = \lim_{s\to 0}\frac{s\times\frac{1}{s^2}}{1+\frac{k}{s}}$$

$$e_{ss} = \frac{1}{k} = \frac{1}{10} = 0.1$$

Hence, the correct answer is 0.1.

Key Point

Calculation of error coefficient using bode plot :

Error coefficient method is used to determine the steady state furor of CLTF by using OLTF.

(i) For Type -1 System.

Initial slope for Type -1 System is -20 dB/dec

This slope will intersect 0 dB line at

$$\omega = k = k_v$$

(ii) For Type - 2 System.

Initial slope for Type -2 system is -40 dB/dec

This slope will intersect 0 dB line at

$$\omega = \sqrt{k}$$

$$k = k_a = (\omega)^2$$

(iii) For Type - n system.

Initial slope for Type - n system is $-\ 20\frac{n\ \text{dB}}{\text{dec}}$

This slope will intersect 0 dB line at

$$\omega = (k)^{\frac{1}{n}}$$

(iv) When n Number zeros present at origin.

Initial slope for n number of zeros is $20\ \text{n}\frac{\text{dB}}{\text{dec}}$

This slope will intersect 0 dB line at

$$\omega = (k)^{-\frac{1}{n}} \Rightarrow k = \left(\frac{1}{\omega}\right)^{\frac{1}{n}}$$

Method 2

Initial slope is ($-$ 20 dB/deC). It Means that there is one pole at origin.

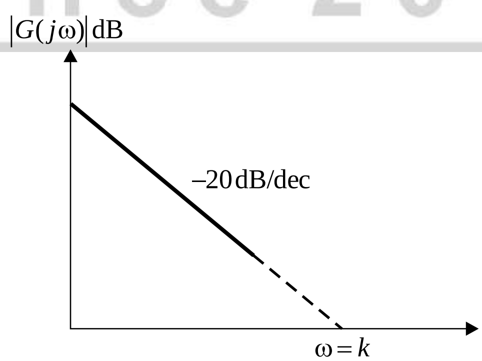


Fig. Type -1 System.

$$\omega = \omega_{gc} = 10\text{rad/sec} = k_v$$

$$k = k_v = 10$$

Steady state error for Type - 1 System for unit ramp input.

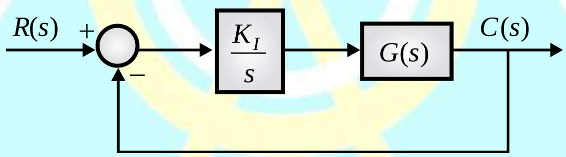
$$e_{ss} = \frac{1}{k_v} = \frac{1}{10} = 0.1$$

Hence the correct answer is 0.1.

Question 59 ***[Control System : Frequency Response]***

Consider the stable closed-loop system shown in the figure. The magnitude and phase values of the frequency response of $G(s)$ are given in the table. The value of gain $K_1(>0)$ for a 50^0 phase margin is _______ (rounded off to 2 decimal places).

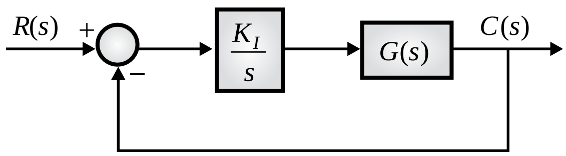
ω in rad/sec	Magnitude in dB	Phase in degrees
0.5	–7	–40
1.0	–10	–80
2.0	–18	–130
10.0	–40	–200



Ans. 1.11 to 1.13

Sol. Given :

ω in rad/sec	Magnitude in dB	Phase in degrees
0.5	–7	–40
1.0	–10	–80
2.0	–18	–130
10.0	–40	–200



Open loop Transfer $= \left(\frac{K_I}{s}\right) G(s)$

$$G'(s) = \left(\frac{k_I}{s}\right) G(s)$$

Put $S = j\omega$

$$G'(j\omega) = \frac{k_I}{j\omega} G(j\omega)$$

Phase margin is given by.

$$PM = 180^0 \angle G'(j\omega_{gc})$$

$$50^0 = 180^0 + \left[-90^0 + \angle G(j\omega_{gc})\right]$$

$$\angle G(j\omega_{gc}) = -40^0$$

Magnitude at gain cross over frequency is given by

$$|G(j\omega_{gc})H(j\omega_{gc})| = 1$$

Or

$$20\log|G(j\omega_{gc})H(j\omega_{gl})| = 0 \text{ dB}$$

From given table we can write gain cross over frequency.

$$\omega_{gc} = 0.5 \text{ rad/sec}$$

$$20\log|G'(j\omega_c)| = 0 \text{ dB}$$

Or

$$|G'(j\omega_c)| = 1 \quad \dots \text{(i)}$$

$$\left|\frac{k_I}{\omega}\right| |G(j\omega_C)| = 1$$

$$20\log|G(j\omega_g)| = -7$$

$$\log_{10}|G(j\omega_{gc})| = (-0.35)$$

$$|G(j\omega_{gc})| = 0.447$$

From equation (i)

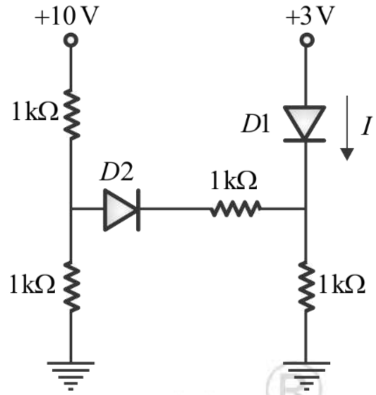
$$\left|\frac{k_I}{0.5}\right| (0.447) = 1$$

$$k_I = 1.12$$

Hence the correct answer is 1.12.

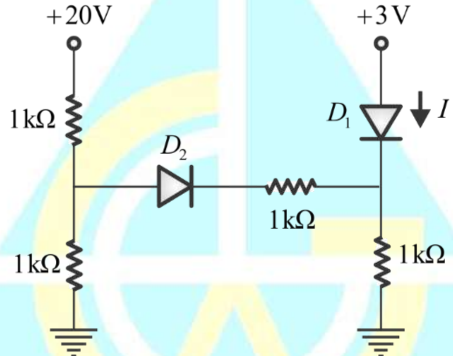
Question 60 ***[Analog Electronics : Diode Family]***

In the given circuit, the diodes are ideal. The current I through the diode $D1$ in miliamperes is ________ (rounded off to 2 decimal places).



Ans. **1.64 to 1.70**

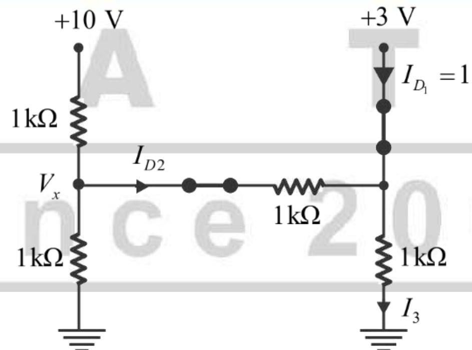
Sol. Given circuit is shown below.



Case 1 :

(i) Assume both diodes D_1 and D_2 are ON.

(ii) For both diode to be ON. I_{D_1} and I_{D_2} should be greater than zero from p to n.



Applying nodal analysis at node x.

$$\frac{V_x - 10}{1\,\text{k}\Omega} + \frac{V_x}{1\,\text{k}\Omega} + \frac{V_x - 3}{1\,\text{k}\Omega} = 0$$

$$3V_x - 13 = 0$$

$$V_x = \frac{13}{3} = 4.33 \text{ volt}$$

Current flowing diode D_2 is I_{D_2}.

$$I_{D_2} = \frac{V_x - 3}{1\text{ k}\Omega} = \frac{4.33 - 3}{1\text{ k}\Omega}$$

$$I_{D_2} = 1.33\text{mA}$$

$$\text{Current } (I_3) = \frac{3}{1\ k\Omega} = 3\text{ mA}$$

Applying KCL :

$$I_{D_2} + I_{D_1} = 3\text{mA}$$

$$I_{D_1} = I = (3\text{mA} - 1.33\text{mA})$$

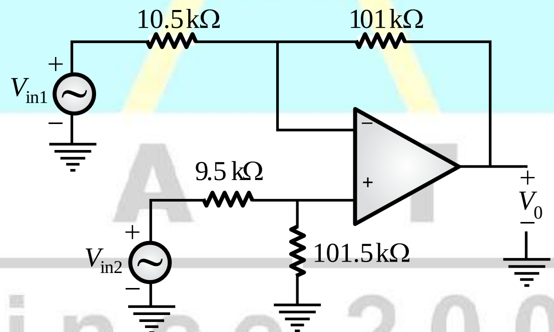
$$I_{D_1} = I = 1.67\text{ mA}$$

Since I_{D_1} and I_{D_2} are greater than zero. Our whole assumption is correct.

Hence the correct answer is 1.67.

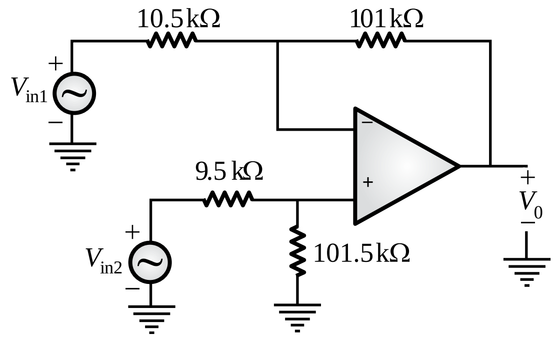
Question 61 ***[Analog Electronics : OP-Amp]***

A difference amplifier is shown in the figure. Assume the op-amp to be ideal. The CMRR (in dB) of the difference amplifier is ________ (rounded off to 2 decimal places).



Ans. **39.50 to 41.50**

Sol. Given Circuit is shown below.



Redraw above Circuit diagram:

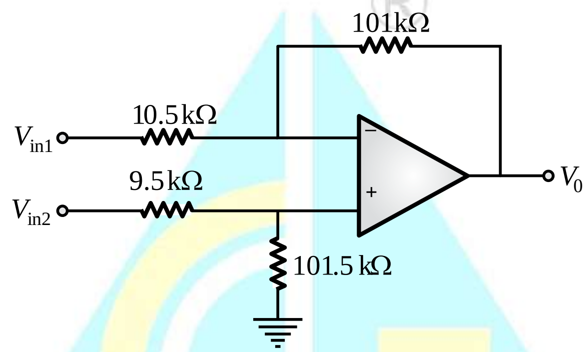


Fig. Difference Amplifier

Output voltage

$$V_0 = V_2\left[\frac{R_4}{R_3+R_4}\right]\left[\frac{R_1+R_2}{R_1}\right] - V_1\left(\frac{R_2}{R_1}\right)$$

$$V_0 = \left(\frac{101.5}{101.5+9.5}\right)\left(\frac{101+10.5}{10.5}\right)V_2 - V_1\left(\frac{101}{10.5}\right)$$

$$V_0 = \underbrace{9.71}_{A_1}V_{in2} - \underbrace{9.619}_{A_2}V_{in1}$$

Common mode rejection ratio (CMRR)

$$CMRR = \left|\frac{Ad}{Ac}\right|$$

Where :

$Ad =$ Differential mode gain

$Ac =$ Common mode gain

Differential Mode gain is given by.

$$A_{DM} = \frac{A_1 - A_2}{2} = \frac{9.71-(-9.619)}{2}$$

$$A_{DM} = 9.6645$$

Common mode gain is given by

$$A_{CM} = A_1 + A_2$$

$= 9.71 - 9.619$

$A_{CM} = 0.091$

$$CMPR = \left|\frac{9.6645}{0.091}\right|$$

$= 106.2$

CMRR (In dB) = 20 log (106 .2)

CMRR (In dB) = $40.52\,\text{dB}$

Hence the correct Answer is $40.52\,\text{dB}$

Question 62 **[Power Electronics : 1-Reactifier]**

A single-phase half-controlled bridge converter supplies an inductive load with ripple free load current. The triggering angle of the converter is 60^0. The ratio of the rms value of the fundamental component of the input current to the rms value of the total input current of the bridge is ________ (rounded off to 3 decimal places).

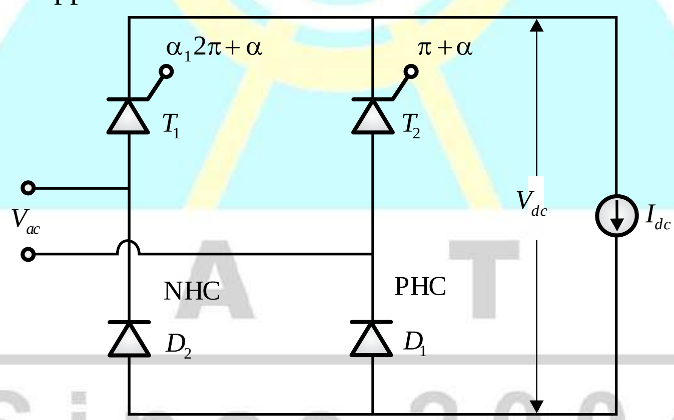
Ans. 0.940 to 0.970

Sol. Given :

(i) A Half - controlled single phase converter.

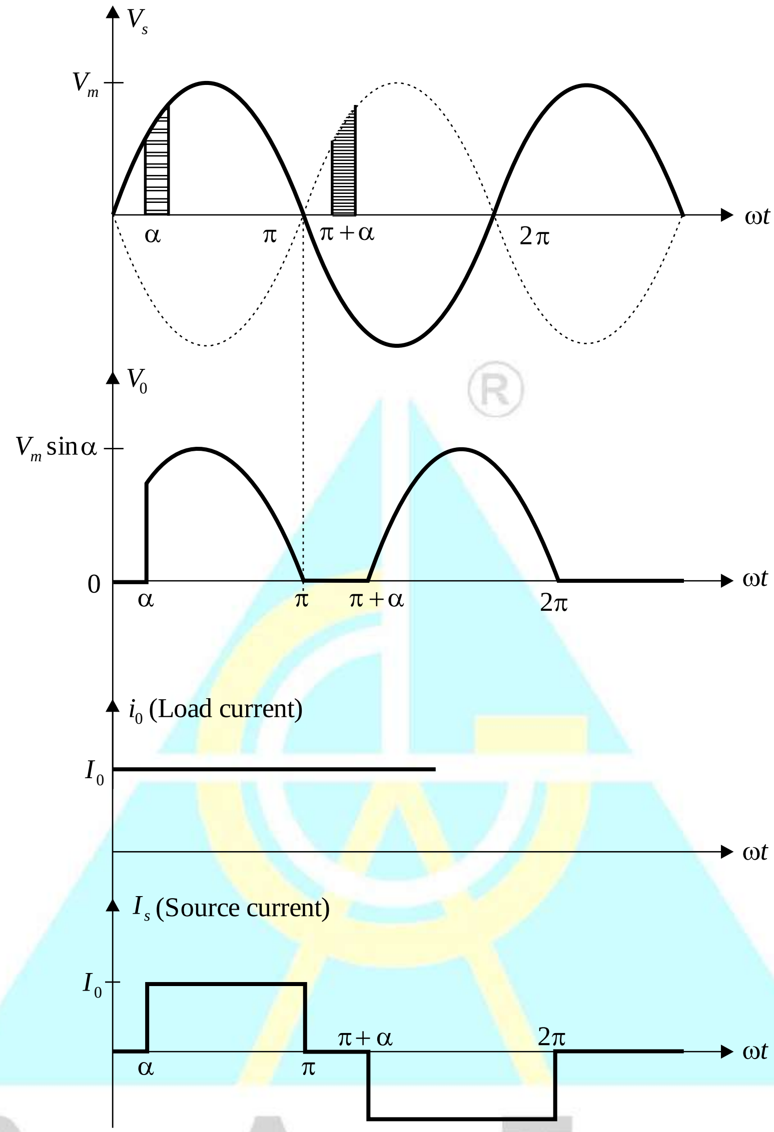
(ii) Firing angle $= 60^0$.

(iii) Inductive load with ripple free load current.



The current source is constant and unidirectional. This implies highly inductive load.

waveform :



- Average value of source current

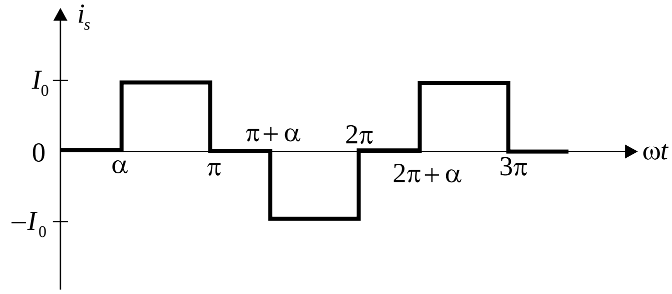
$i_{s(\text{ avg })} = 0$ [From the waveform]

- Rms value of source current.

$$I_{S(Rms)} = I_0\sqrt{\frac{(\pi-\alpha)}{(\pi+\alpha)-\alpha}}$$

$$I_{S(Rms)} = I_0\sqrt{\frac{(\pi-\alpha)}{\pi}}$$ [From the waveform]

- Quasi-square waveform



Fourier series representation:

$$i_s(t) = \sum_{n=1,3,5}^{\infty} \frac{4I_0}{n\,\pi} \cos\left(\frac{n\alpha}{2}\right) \sin\left(n\,\omega t - \frac{n\alpha}{2}\right)$$

Rms Value of n^{th} Harmonic of $i_s(t)$

$$I_{s\gamma_n} = \frac{c_n}{\sqrt{2}} = \frac{2\sqrt{2}}{n\pi} I_0 \cos\left(\frac{n\alpha}{2}\right)$$

Rms value of fundamental Harmonic of $i_s(t)$.

$$I_{sr_1} = \frac{2\sqrt{2}}{\pi} \cos\left(\frac{\alpha}{2}\right) I_0$$

Input current distortion factor (CDF):

$$CDF = \frac{I_{sr_1}}{I_{sr}} = \frac{\frac{2\sqrt{2}}{\pi} I_0 \cos\left(\frac{\alpha}{2}\right)}{I_0\sqrt{\frac{(\pi-\alpha)}{\pi}}}$$

$$CDF = \frac{2\sqrt{2}\cos\left(\frac{\alpha}{2}\right)}{\pi\sqrt{\left(\frac{\pi-\alpha}{\pi}\right)}} = \frac{2\sqrt{2}\cos 30^0}{\pi\sqrt{1-\frac{\pi}{3\pi}}}$$

$$CDF = \frac{2\sqrt{2}(0.866)}{\pi\sqrt{\frac{2}{3}}} = \frac{2\sqrt{3}(0.866)}{\pi}$$

$$CDF = 0.955$$

Hence the correct answer is 0.955

Question 63 ***[Power Electronics : Inverter]***

A single-phase bridge voltage source inverter (VSI) feeds a purely inductive load. The inverter output voltage is a square wave in 180^0 conduction mode. The fundamental frequency of the output voltage is 50 Hz. If the DC input voltage of the inverter is 100 V and the value of the load inductance is 20 mH, the peak-to-peak load current in amperes is _____ (rounded off to the nearest integer).

Ans. 50 to 50

Sol. Given :

(i) Input voltage = 100 V

(ii) Load inductance = 20 mH

(iii) Fundamental frequency of the output voltage is = 50 Hz

is = 5O HZ:

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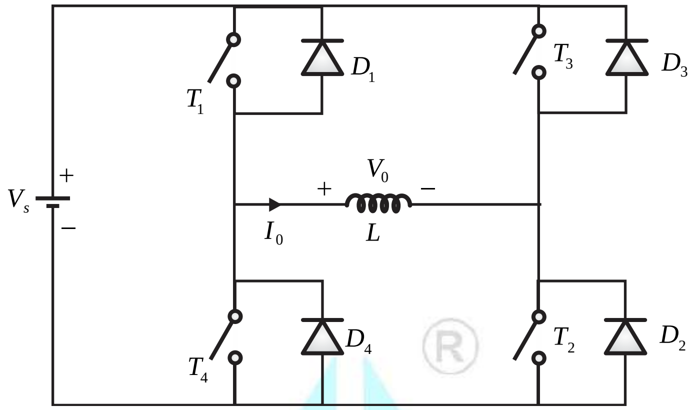
(iv) A single phase bridge voltage source inverter (VSI) feeds a purely inductive load.



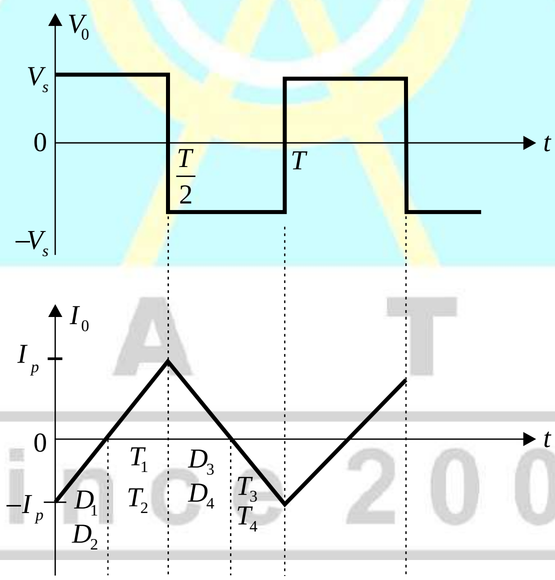
L-Load : I_0 is triangular and symmetric about time - axis and feedback diodes and switches conducts for equal duration i.e $\frac{T}{4}$.

Output voltage wave form :

$$0 < t < T/2$$

$$V_L = V_0 = V_S$$

Let us assume V_0 and I_0 as given in the figure.



Mode : $0 < t < T/2$

$$V_L = V_0 = V_S$$

$$\frac{L[I_p - (-I_p)]}{\left(\frac{T}{2}\right)} = V_S$$

$$\frac{L(2I_p)}{\left(\frac{T}{2}\right)} = V_S$$

$$2I_p = \left(\frac{V_S}{L}\right)\left(\frac{T}{2}\right)$$

$$I_p = \left(\frac{V_s}{L}\right)\left(\frac{T}{4}\right) = \left(\frac{V_s}{L}\right)\left(\frac{1}{4f}\right)$$

$$I_p = \frac{100}{20\times10^{-3}}\left(\frac{1}{4\times50}\right)$$

$$I_p = 25 \text{ Amp.}$$

$$2I_p \text{ (peak to peak Load current) } = 50 \text{ Amp}$$

Hence the correct answer is 50.

Key Point

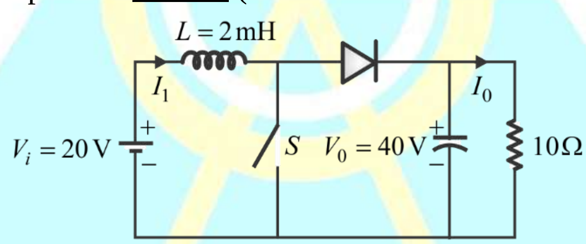
(i) R-Load: Output voltage and output current are of square wave.

(ii) L-Load : Output voltage is a square wave but output current is a triangular wave and output current lags output Voltage by 90^0.

(iii) For Any load in VSI. The output voltage is always

Question 64 ***[Power Electronics : Chopper]***

In the DC-DC converter shown in the figure, the current through the inductor is continuous. The switching frequency is 500 Hz. The voltage (V_0) across the load is assumed to be constant and ripple free. The peak inductor current in amperes is ______ (rounded off to the nearest integer).



Ans. **13 to 13**

Sol. **Given :**

$$V_S = 20 \text{ V}$$
$$L = 2 \text{ mH}$$
$$V_0 = 40 \text{ V}$$
$$f = 500 \text{ Hz}$$
$$R = 10\ \Omega$$

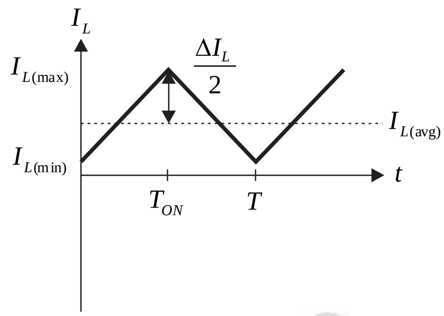
For Boost Converter Average Value of Output Voltage

$$V_0 = \frac{V_S}{(1-\alpha)}$$

$$(1-\alpha) = \frac{20}{40} = \frac{1}{2}$$

$$\alpha = \frac{1}{2} = 0.5$$

Inductor current waveform :



Mode 1 : $0 < t < T_{ON}$

Switch-ON and Diode-OFF

$$V_L = V_S$$

$$\frac{LdI_L}{dt} = V_S$$

$$\frac{L(I_{\max} - I_{\min})}{dt} = V_S$$

$$\Delta I_L = \frac{V_S}{L} T_{ON}$$

$$\Delta I_L = \frac{\alpha V_S}{fL}$$

$$\Delta I_L = \frac{0.5 \times 20 \times 1000}{500 \times 2}$$

$$\Delta I_L = 10 \text{ Amp}$$

From Inductor Current Waveform :

$$\Delta I_L = I_{L(\max)} - I_{L(\min)} \qquad \ldots (1)$$

$$\Delta I_{L(avg)} = \frac{I_{L(\max)} + I_{L(\min)}}{2}$$

$$2I_{L(avg)} = I_{L(\max)} + I_{L(\min)} \qquad \ldots (2)$$

From equation (1) and (2)

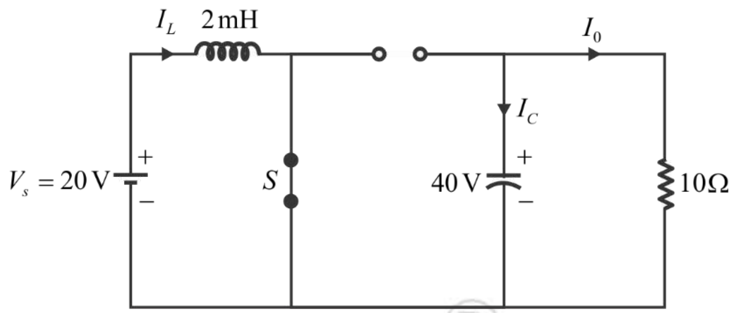
$$\Delta I_L + 2I_{L(avg)} = 2I_{L(\max)}$$

$$I_{L(\max)} = I_{L(avg)} + \frac{\Delta I_L}{2} \qquad \ldots (3)$$

$$I_{L(\min)} = I_{L(avg)} - \frac{\Delta I_L}{2} \qquad \ldots (4)$$

Now for $I_{L(avg)}$: In case of Boost Converter :

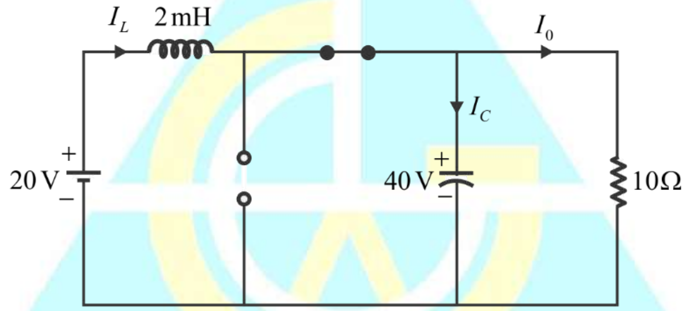
Mode 1 : $0 < t < T_{ON}$



$$I_C + I_0 = 0$$

$$I_C = -I_0 \qquad \ldots (5)$$

Mode 2 : $T_{ON} < t < T$



$$I_L = I_C + I_0$$

$$I_C = I_L - I_0 \qquad \ldots (6)$$

Average Inductor Current :

(·) Use Ampere-Sec balance Across C

$$\int_{<T>} I_C dt = 0$$

$$(-I_0)T_{ON} + (I_L - I_0)T_{OFF} = 0$$

$$(I_L - I_0)T_{OFF} = I_0 T_{ON}$$

$$(I_L - I_0)\ (1-\alpha)T = I_0 \propto T$$

$$(I_L - I_0) = \frac{\alpha}{(1-\alpha)} I_0$$

$$I_{L(avg)} = \frac{\alpha}{(1-\alpha)} I_0 + I_0$$

$$I_{L(avg)} = \frac{\alpha I_0 + I_0 - \alpha I_0}{(1-\alpha)}$$

$$I_{L(avg)} = \frac{I_0}{1-\alpha}$$

When Diode is ON. During $T_{ON} < t < T$. Inductor current goes to L.

$$I_0 = \frac{I_L T_{OFF}}{T} = \frac{I_L (1-\alpha)T}{T}$$

$$I_{L(avg)} = \frac{I_0}{(1-\alpha)}$$

Now,

$$I_{L(avg)} = \frac{V_0}{R(1-\alpha)} = \frac{40}{10\times 0.5}$$

$$= \frac{4}{0.5} = 8 \text{ Amp}$$

$$I_{L(avg)} = 8 \text{ Amp}$$

The peak Inductor Current

$$I_{L(\max)} = I_{L(avg)} + \frac{\Delta I_L}{2}$$

$$= 8 + \frac{10}{2}$$

$$I_{L(\max)} = 13 \text{ Amp}$$

Question 65 ***[Power Electronics : Rectifier]***

A single-phase full-controlled thyristor converter bridge is used for regenerative braking of a separately excited DC motor with the following specifications :

Rated armature voltage	210 A
Rated armature current	10 A
Rated speed	1200 rpm
Armature resistance	1Ω
Input to the converter bridge	240 V at 50 Hz
The armature of the DC motor is fed from the full-controlled bridge and the field current is kept constant.	

Assume that the motor is running at 600 rpm and the armature terminals of the motor are suitably reversed for regenerative braking. If the armature current of the motor is to be maintained at the rated value, the triggering angle of the converter bridge in degrees should be ________ (rounded off to 2 decimal places).

Ans. 113.00 to 116.00

Sol. Given :

(i) Rated armature voltage $= 210\,\text{A}$

(ii) Rated armature current $= 10\,\text{A}$

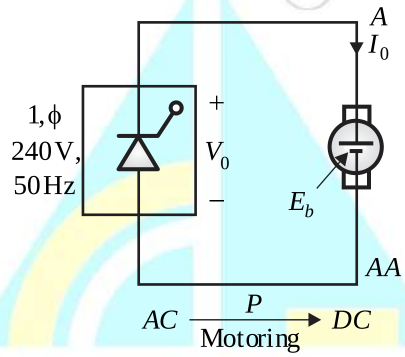
(iii) Rated speed $= 1200\,\text{rpm}$

(iv) Armature resistance $= 1\,\Omega$

(v) Input to the converter bridge $= 240\,\text{V}$ at $50\,\text{Hz}$

(vi) The armature of the DC motor is fed from the full-controlled bridge and the field current is kept constant.

(vii) Assume that the motor is running at 600 rpm.



Under rated operating conditions at the separately excited dc motor.

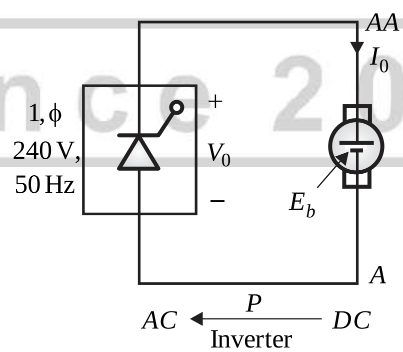
$$V_t = E_a + I_a r_a$$

$$V_t = K_m w_m + I_a r_a$$

$$V_t = K_m\left(\frac{2\pi N}{60}\right) + I_a r_a$$

$$210 = K_m\left(\frac{2\pi.1200}{60}\right) + (10\times 1)$$

$$K_m = 1.59\ \text{v}-\text{s/rad}\ \text{(or)}\ 1.59\ \ \text{N}-\text{m/A}$$



Now according to question. Armature terminals reversed

$$E_b \rightarrow (\text{Negative})$$

$$V_0 = V_t = -E_b + I_a r_a$$

$$\frac{2V_m}{\pi}\cos\alpha = -E_b + I_a r_a$$

$$\frac{2\left(240\sqrt{2}\right)}{\pi}\cos\alpha = -(1.59)\left(\frac{2\pi\times 600}{60}\right)+10$$

$$216.0759\cos\alpha = -99.90+10$$

$$\cos\alpha = -\frac{89.9}{216.076}$$

$$\cos\alpha = -0.416$$

$$\alpha = 114.58^0$$

Hence the correct answer is 114.58.

❖❖❖❖